

Curs 3

2019/2020

# Dispozitive și circuite de microunde pentru radiocomunicații

# Disciplina 2019/2020

- 2C/1L, **DCMR (CDM)**
- Minim 7 prezente (curs+laborator)
- Curs - **conf. Radu Damian**
  - Marti 14-15, P7
  - E – **50%** din nota
  - probleme + (2p prez. curs) + (3 teste) + (bonus activitate)
    - primul test L1 (t2 si t3 neanuntate)
    - 3pz (C) = +0.5p
  - toate materialele permise

# Disciplina 2019/2020

- 2C/1L, **DCMR (CDM)**
- Laborator – **conf. Radu Damian**
  - Miercuri 10-14 impar II.12 (par eng.)
  - Joi 14- 16 par
  - L – **25%** din nota
    - prezenta + rezultate personale
  - P – **25%** din nota
    - tema personala

# Documentatie

■ <http://rf-opto.etti.tuiasi.ro>

Laboratorul de Microunde si Opti

Not secure | rf-opto.etti.tuiasi.ro/microwave\_cd.php?chg\_lang=1

English | Romana

Start **Didactic** Master Colectiv Cercetare Studenti Admin

Microunde Comunicatii Optice Optoelectronica Internet Antene Practica Retele Soft didactic

## Dispozitive si circuite de microunde pentru radiocomunicatii

### Disciplina: DCMR (2017-2018)

**Coordonator Disciplina:** conf. dr. Radu-Florin Damian

**Cod:** DOS412T

**Tip Disciplina:** DOS; Disciplina Optionala, Disciplina de Specialitate

**Credite:** 4

**An de Studiu:** 4, Sem. 7

### Activitati

**Curs:** Cadru Didactic: conf. dr. Radu-Florin Damian, 2 Ore/Saptamana, Sectie Specializare, Orar:

**Laborator:** Cadru Didactic: conf. dr. Radu-Florin Damian, 1 Ore/Saptamana, Grupa, Orar:

### Evaluare

Tip: **Examen**

**A:** 50%, (Examen/Colocviu)

**B:** 25%, (Activitate Seminar/Laborator/Proiect)

**D:** 25%, (Teme de casa/Lucrari de specialitate)

### Note

[Rezultate totale](#)

### Prezenta

[Curs](#)

[Laborator](#)

### Liste

[Bonus-uri acumulate \(final\)](#)

[Studenti care nu pot intra in examen](#)

# Bibliografie

- <http://rf-opto.etti.tuiasi.ro>
- Irinel Casian-Botez: "Microunde vol. 1: Proiectarea de circuit", Ed. TEHNOPRES, 2008
- David Pozar, Microwave Engineering, Wiley; 4th edition , 2011, ISBN : 978-1-118-29813-8 (E), ISBN : 978-0-470-63155-3 (P)

# Fotografii

Nr. Student	Prezent	Nr. Student	Prezent	Nr. Student	Prezent
1 ANGHIELUS IONUT-MARIUS	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	2 ANTIGHIN FLORIN-RAZVAN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	3 ANTONICA BIANCA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
4 APOSTOL PAVEL-MANUEL	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	5 BALASCA IULIAN-PETRU	<input checked="" type="checkbox"/> Puncte: 0 Nota: 0 Obs:	6 BOSTAN ANDREI-PETRICIA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
7 BOTEZAT EMANUEL	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	8 BUTUNOI GEORGE-MADALIN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	9 CHILEA SALUCA-MARIA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
10 CHERITOU ECATERINA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	11 COJOC MARIUS	<input checked="" type="checkbox"/> Puncte: 0 Nota: 0 Obs:	12 COJOCARI AURA-FLORINA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:

Nr. Student	Prezent
2 ANTIGHIN FLORIN-RAZVAN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:

# Acces

## ■ Personalizat



### Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

[Acceseaza ca acest student](#)

### Note obtinute

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
<b>TW</b>	<b>Tehnologii Web</b>					
N		17/01/2014	Nota finala	10	-	
A		17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
B		17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
D		17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

Nume

Email

Cod de verificare

Trimite

# Examen: Reprezentare logaritmică

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 $\mu$ W
-30 dBm	= 1 $\mu$ W
-60 dBm	= 1 nW

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

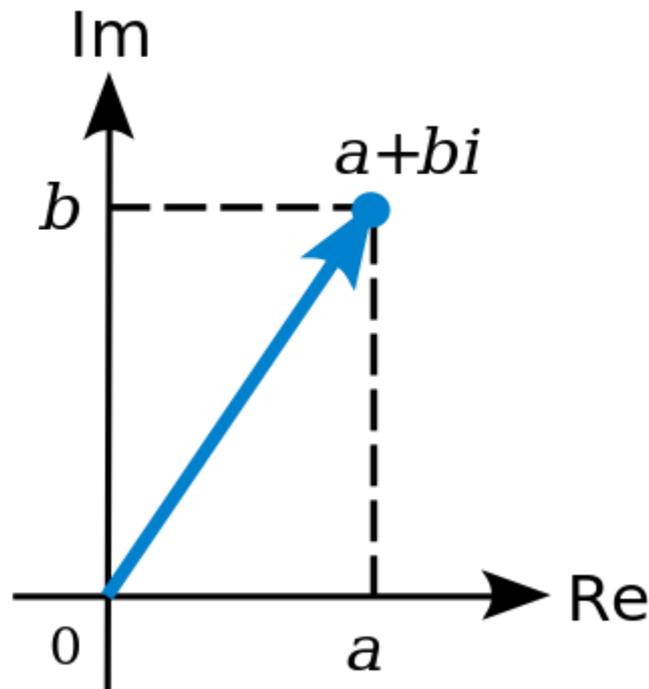
$$[\text{x}] + [\text{dB}] = [\text{x}]$$

# Examen

- Operatii cu numere complexe!
- $z = a + j \cdot b ; j^2 = -1$

# Plan complex

- abscisa – partea reala
- ordonata – partea imaginara
- oricare poate fi negativa, intregul plan, 4 cadrane



# Reprezentare polara

## ■ Reprezentare polara

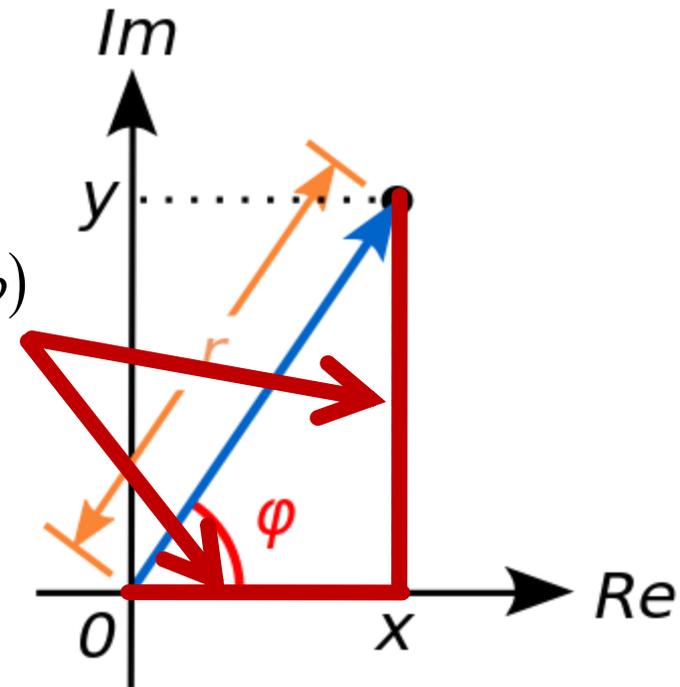
■ modul

■ faza

$$z = a + j \cdot b = |z| \cdot (\cos \varphi + j \cdot \sin \varphi)$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\varphi = \arg(z) = \begin{cases} \arctan\left(\frac{b}{a}\right), & a > 0 \\ \arctan\left(\frac{b}{a}\right) + \pi, & a < 0, b \geq 0 \\ \arctan\left(\frac{b}{a}\right) - \pi, & a < 0, b < 0 \\ \frac{\pi}{2}, -\frac{\pi}{2}, \text{ nedefinit} & a = 0 \end{cases}$$



# Introducere

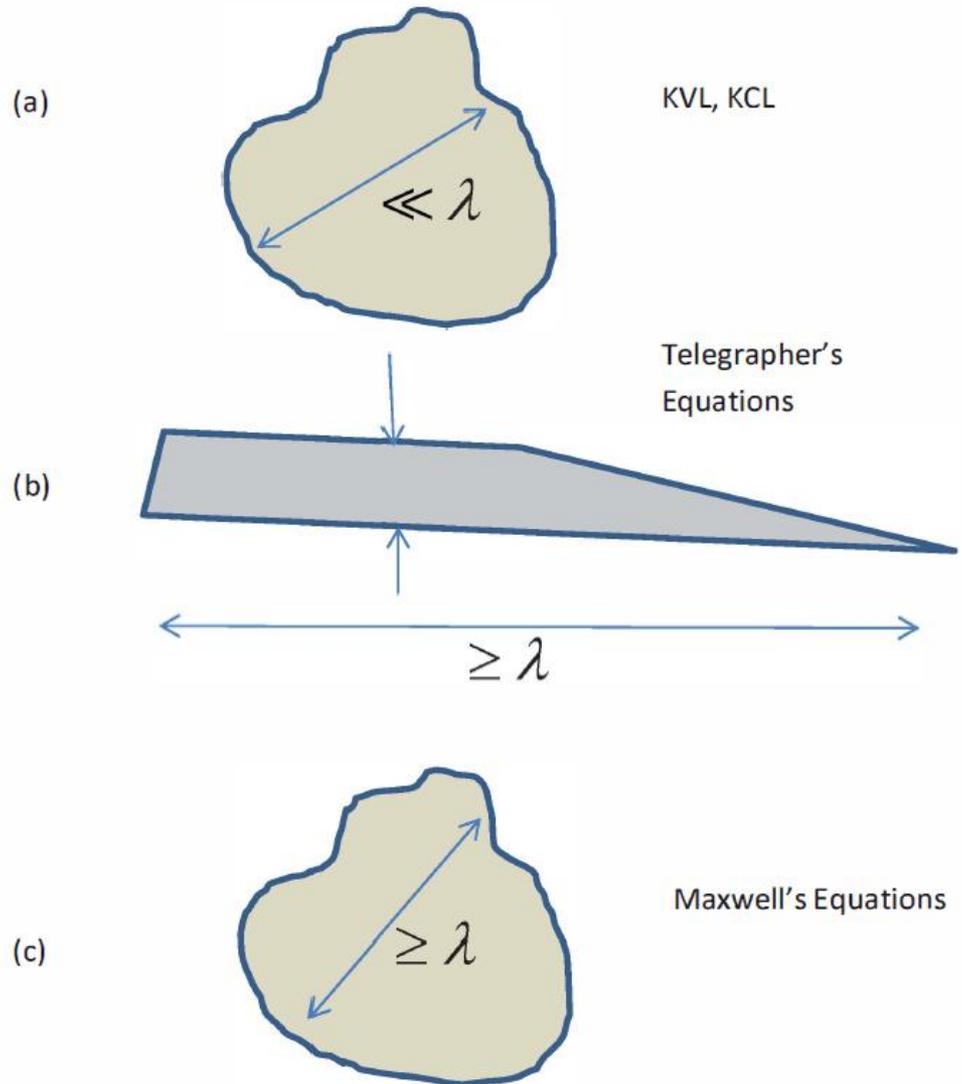
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# Lungime electrica

- Comportarea (descrierea) unui circuit depinde de lungimea sa electrica la frecventele de interes

- $E \approx 0 \rightarrow$  Kirchhoff
- $E > 0 \rightarrow$  propagare

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda}\right)$$



# Ecuatiile lui Maxwell

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

## ■ Ecuatii constitutive

$$D = \varepsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

## • In vid

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\varepsilon_0 = 8,854 \times 10^{-12} \text{ F/m}$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ m/s}$$

# Ecuatiile de propagare

- Ecuatiile Helmholtz sau ecuațiile de propagare

Mediu lipsit de sarcini electrice

$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \epsilon \mu + j\omega \mu \sigma$$

$\gamma$  – Constanta de propagare

# Solutia ecuatiilor de propagare

$E_y = E^+ e^{-\gamma \cdot z} + E^- e^{\gamma \cdot z}$  Camp electric dupa directia Oy, ← prin alegere judicioasa  
propagare dupa directia Oz ← a sistemului de referinta

$$\gamma = \sqrt{-\omega^2 \epsilon \mu + j \omega \mu \sigma} = \alpha + j \cdot \beta$$

- unda
  - incidenta
  - reflectata
- unda
  - directa
  - inversa

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)}$$

$$(\omega \cdot t - \beta \cdot z) = \text{const}$$

$$E_y = E^- \cdot e^{\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

$$(\omega \cdot t + \beta \cdot z) = \text{const}$$

punctele  
de faza  
constanta:

# Solutia ecuatiilor de propagare

- unda
  - incidenta
  - reflectata
- unda
  - directa
  - inversa

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

$$H_z = H^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + H^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

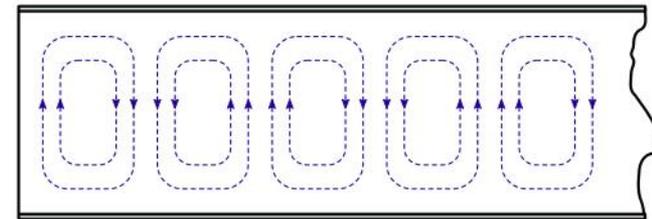
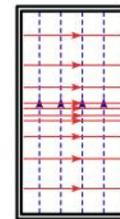
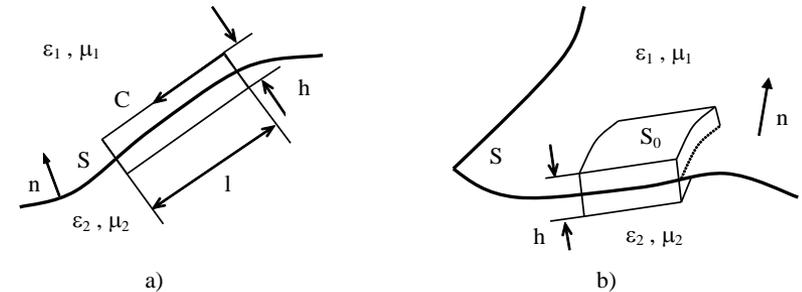
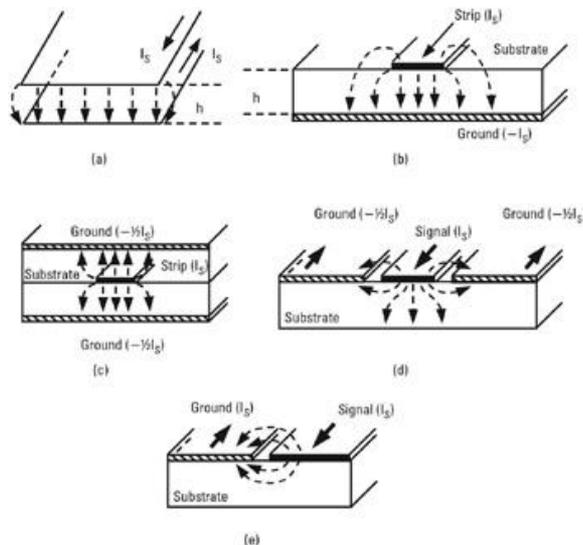
$$V(z) = V^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + V^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

$$I(z) = I^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + I^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

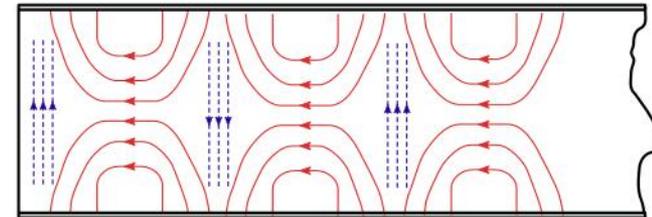
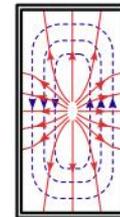
$$V(z) = V^+ \cdot e^{j(\omega t - \beta \cdot z)} + V^- \cdot e^{j(\omega t + \beta \cdot z)}$$

# Moduri in medii delimitate

- Campul electric **trebuie** sa fie perpendicular pe un perete metalic sau nul
- Campul magnetic **trebuie** sa fie tangent la un perete metalic sau nul



TE<sub>10</sub>



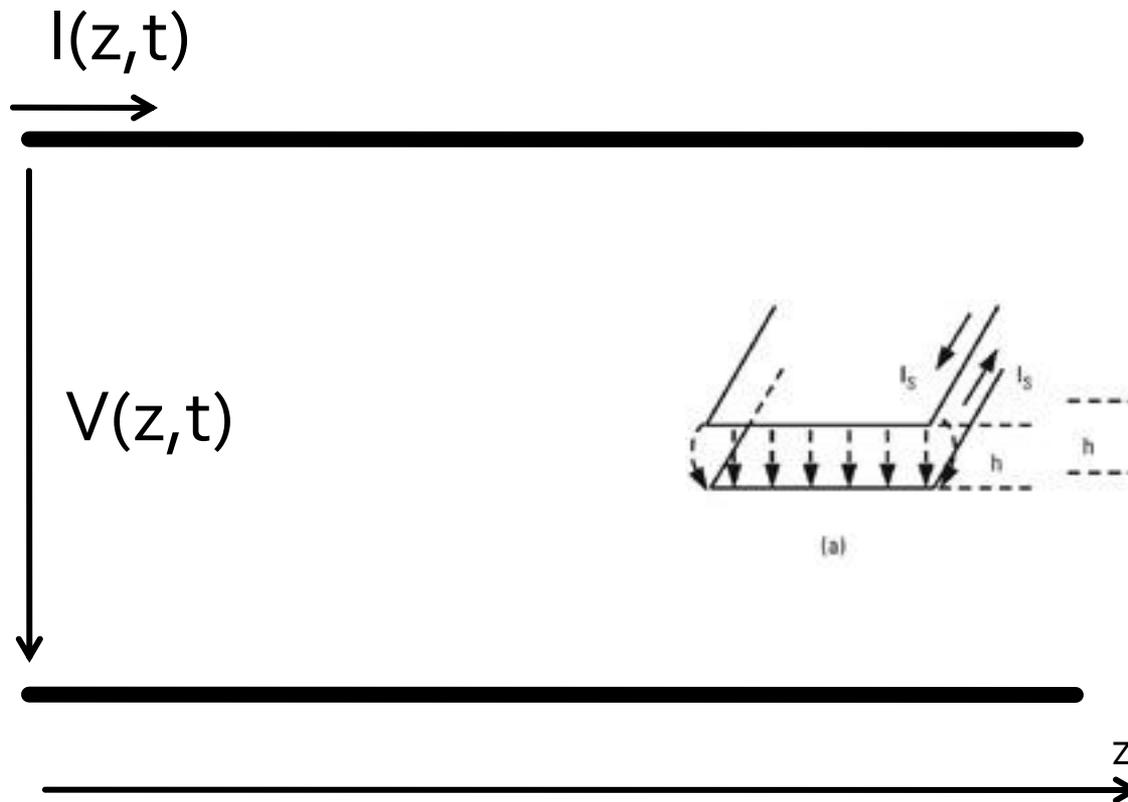
TM<sub>11</sub>

# Linii de transmisie in mod TEM

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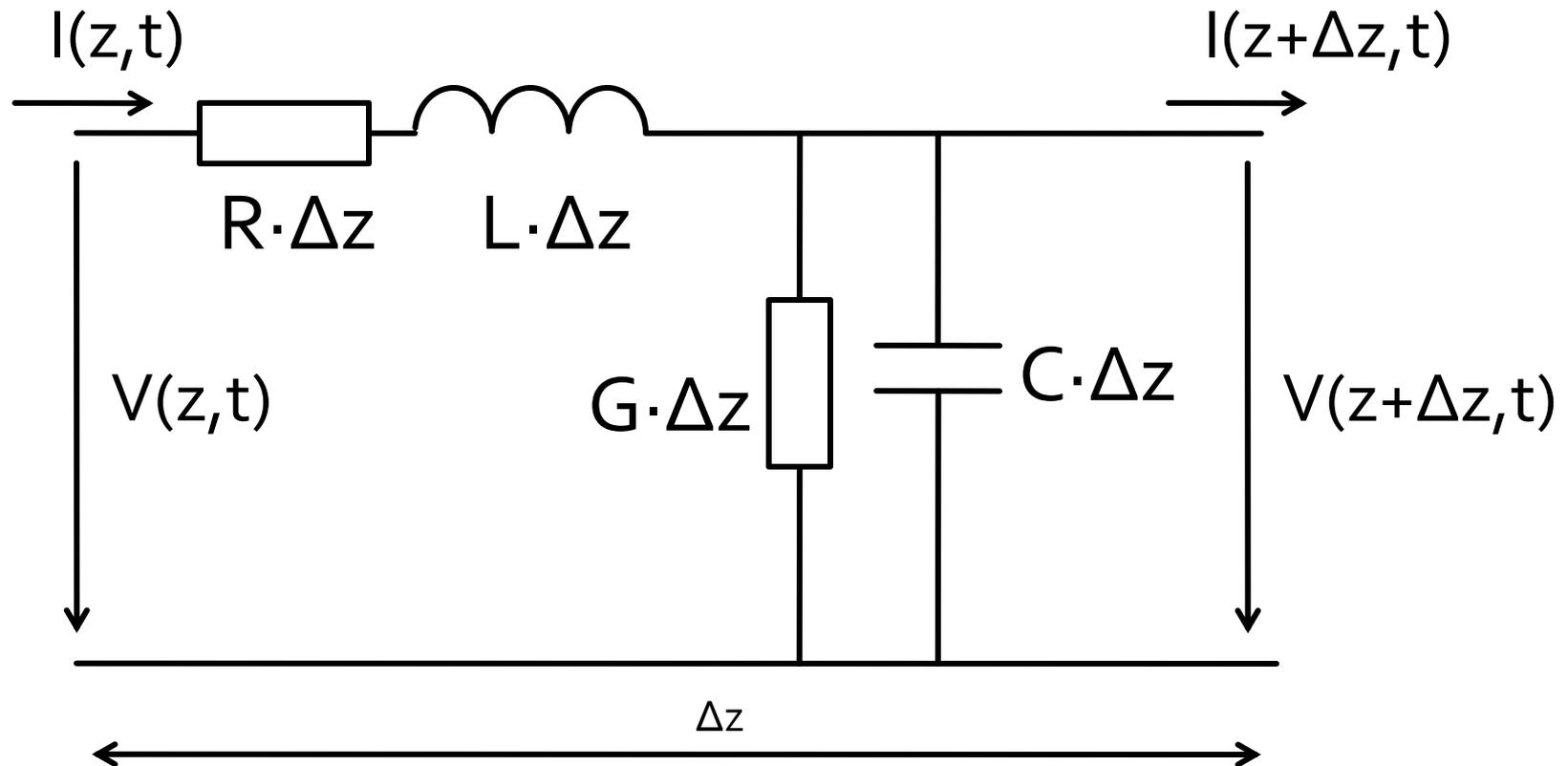
# Linie de transmisie

- mod TEM, doi conductori



# Linie de transmisie model echivalent

- mod TEM, doi conductori



# Ecuatiile telegrafistilor

- domeniu timp

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \quad \text{K II}$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \quad \text{K I}$$

- semnale sinusoidale

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

$$\left/ \frac{d}{dz} (\dots) \right.$$

# Rezolvare

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$


$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma^2 = -\omega^2 \epsilon \mu + j \omega \mu \sigma$$

# Solutiile

$$\left\{ \begin{array}{l} V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z} \\ I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z} \end{array} \right. \quad \gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} (V_0^+ e^{-\gamma \cdot z} - V_0^- e^{\gamma \cdot z})$$

- Impedanta caracteristica a liniei

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \quad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

# Linie fara pierderi

- **Fara pierderi:**  $R=G=0$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)} = j \cdot \omega \cdot \sqrt{L \cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

$$Z_0 = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}} = \sqrt{\frac{L}{C}} \quad \blacksquare \quad Z_0 \text{ real}$$

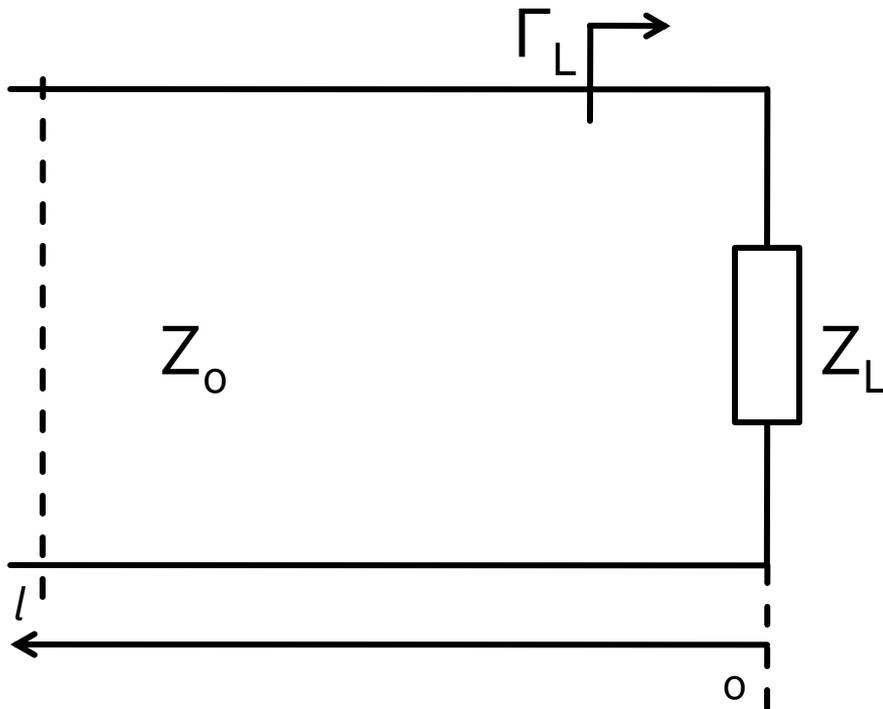
$$V(z) = V_0^+ e^{-j \cdot \beta \cdot z} + V_0^- e^{j \cdot \beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j \cdot \beta \cdot z} - \frac{V_0^-}{Z_0} e^{j \cdot \beta \cdot z}$$

$$\lambda = \frac{2\pi}{\omega \cdot \sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

# Linie fara pierderi



$$V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- coeficient de reflexie in tensiune

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

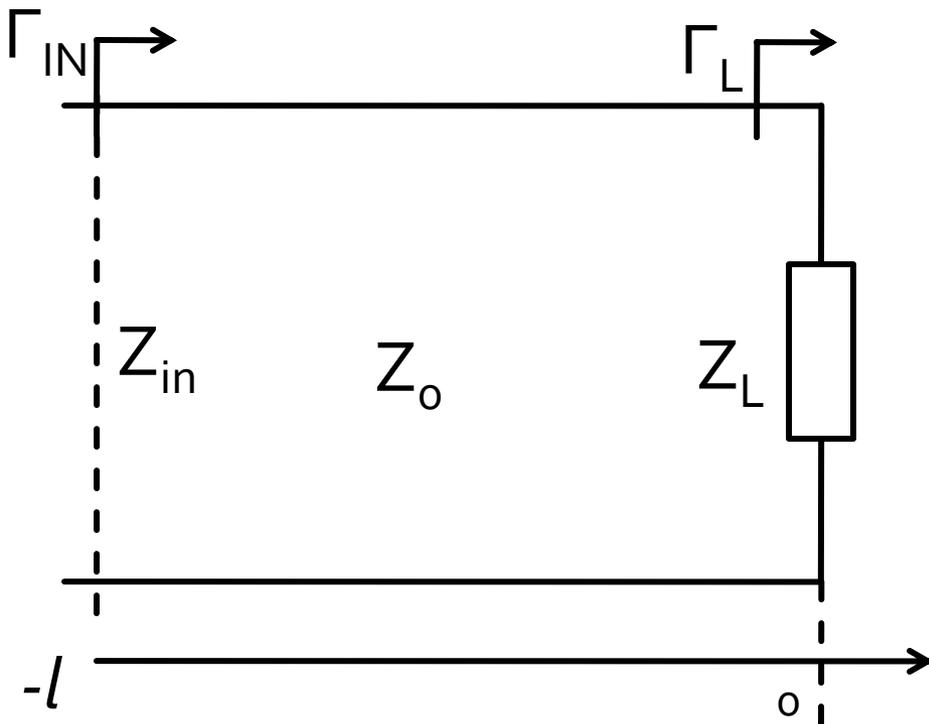
- $Z_0$  real

# Linie fara pierderi

- coeficientul de reflexie la intrarea liniei

$$V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^- \quad \Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\beta \cdot l} + V_0^- e^{-j\beta \cdot l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\beta \cdot l}}{V_0^+ \cdot e^{j\beta \cdot l}} = \Gamma(0) \cdot e^{-2j\beta \cdot l}$$

$$|\Gamma(-l)| = |\Gamma(0)| \cdot |e^{-2j\beta \cdot l}| = |\Gamma(0)|$$

$$\Gamma_{IN} = \Gamma_L \cdot e^{-2j\beta \cdot l}$$

$$|\Gamma_{IN}| = |\Gamma_L|$$

# Linie fara pierderi

$$V(z) = V_0^+ \cdot (e^{-j\beta \cdot z} + \Gamma \cdot e^{j\beta \cdot z})$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta \cdot z} - \Gamma \cdot e^{j\beta \cdot z})$$

## ■ Puterea medie

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\{1 - \Gamma^* \cdot e^{-2j\beta \cdot z} + \Gamma \cdot e^{2j\beta \cdot z} - |\Gamma|^2\}$$

  
(z - z\*) = Im

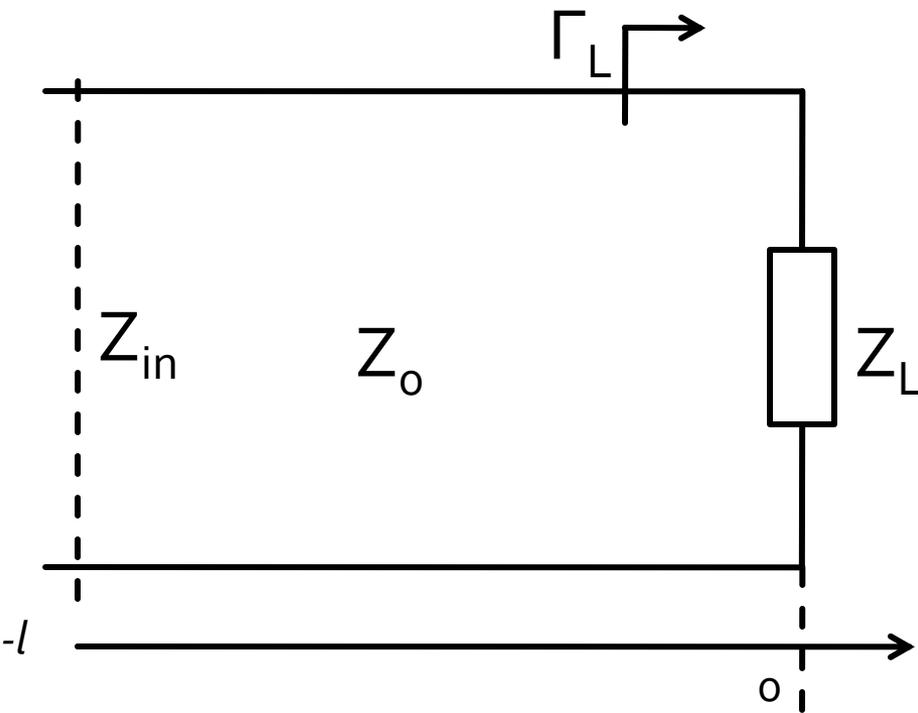
$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

- Puterea transmisa sarcinii = Puterea incidenta - Puterea "reflectata"

- Return Loss [dB]

$$RL = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

# Linie fara pierderi



$$V(-l) = V_0^+ e^{j\beta \cdot l} + V_0^- e^{-j\beta \cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\beta \cdot l} - \frac{V_0^-}{Z_0} e^{-j\beta \cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \quad Z_{in} = Z_0 \cdot \frac{1 + \Gamma \cdot e^{-2j\beta \cdot l}}{1 - \Gamma \cdot e^{-2j\beta \cdot l}}$$

- impedanta la intrarea liniei

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0) \cdot e^{j\beta \cdot l} + (Z_L - Z_0) \cdot e^{-j\beta \cdot l}}{(Z_L + Z_0) \cdot e^{j\beta \cdot l} - (Z_L - Z_0) \cdot e^{-j\beta \cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

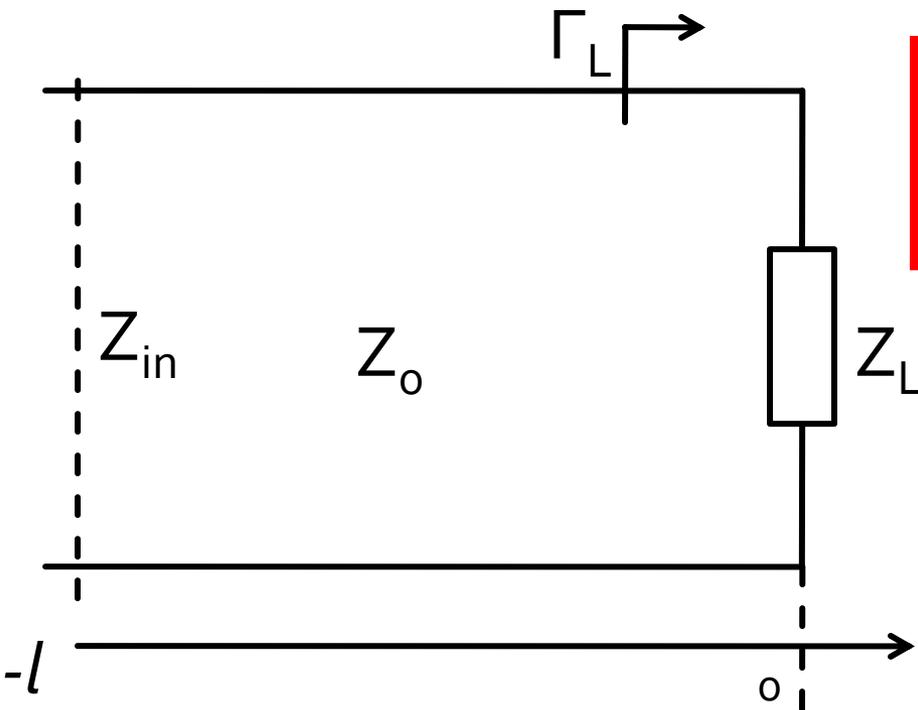
# Linie fara pierderi

- impedanta la intrarea liniei

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# Linie fara pierderi

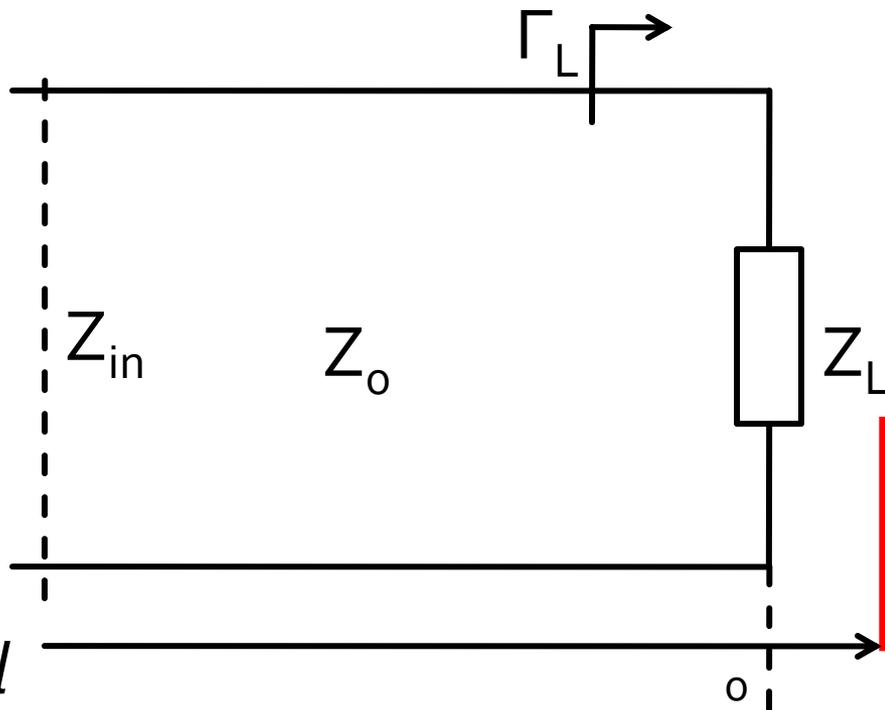
- impedanta la intrarea liniei de impedanta caracteristica  $Z_0$ , de lungime  $l$ , terminata cu impedanta  $Z_L$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# Linie fara pierderi

- relatia este **dependenta de frecventa** prin valoarea  $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \quad \lambda = \frac{2\pi}{\beta}$$
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

dependenta de frecventa este **periodica**, impusa de functia tangenta

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

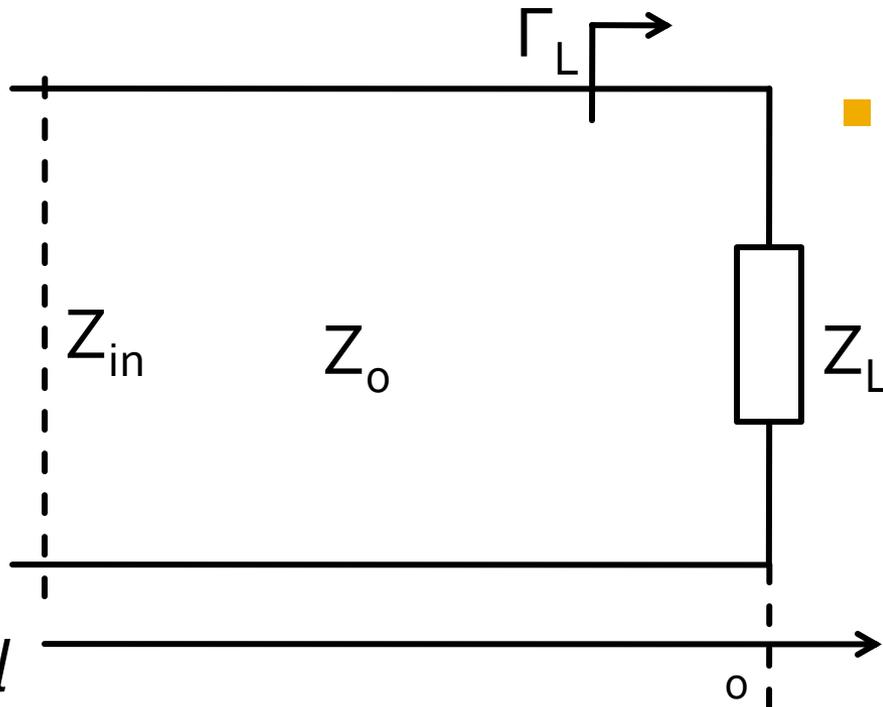
# Linie fara pierderi, cazuri particulare

- $l = k \cdot \lambda / 2$       $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$       $\tan \beta \cdot l = 0$

$$Z_{in} = Z_L$$

- $l = \lambda / 4 + k \cdot \lambda / 2$       $\tan \beta \cdot l \rightarrow \infty$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$



- Transformatorul in sfert de lungime de unda

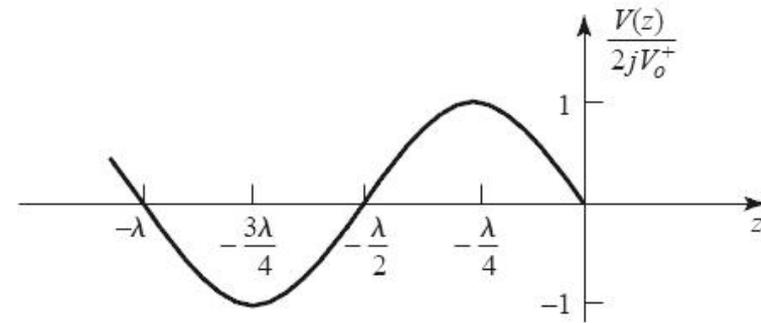
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# Linie in scurtcircuit

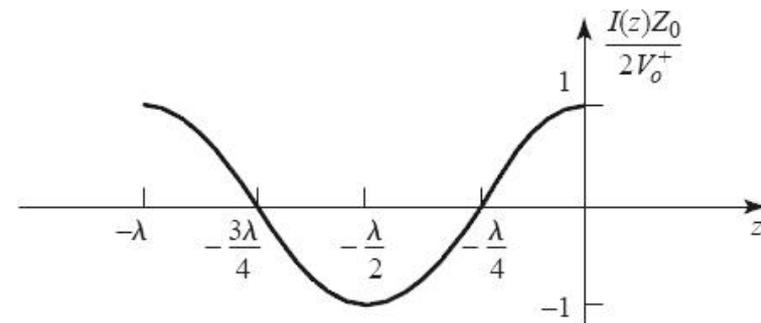
- $Z_L = 0$
- reactanța pură
  - +/- → in funcție de  $l$

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

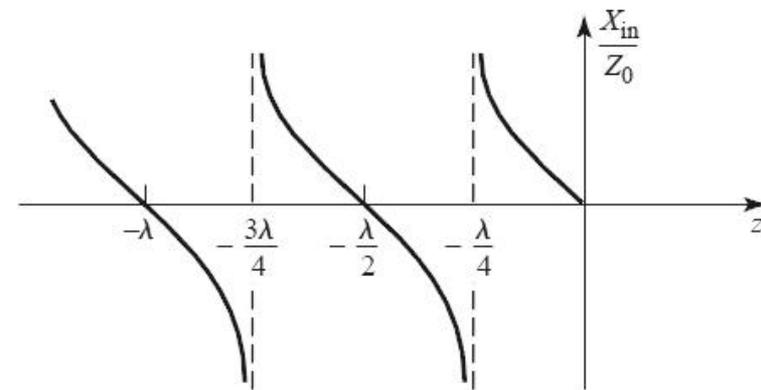
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



(a)



(b)



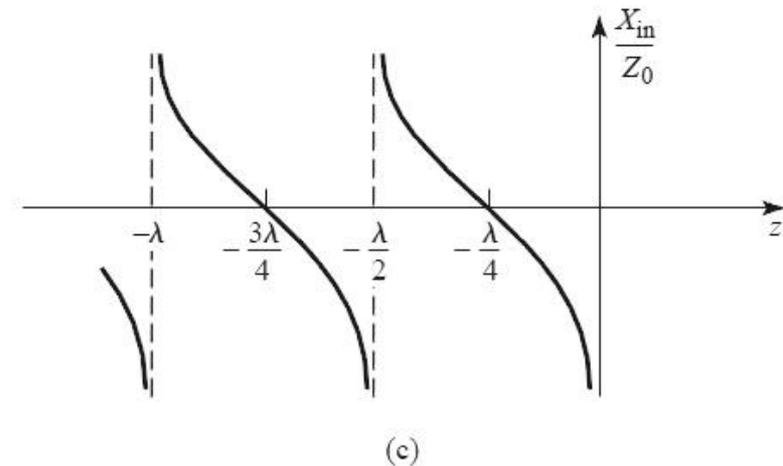
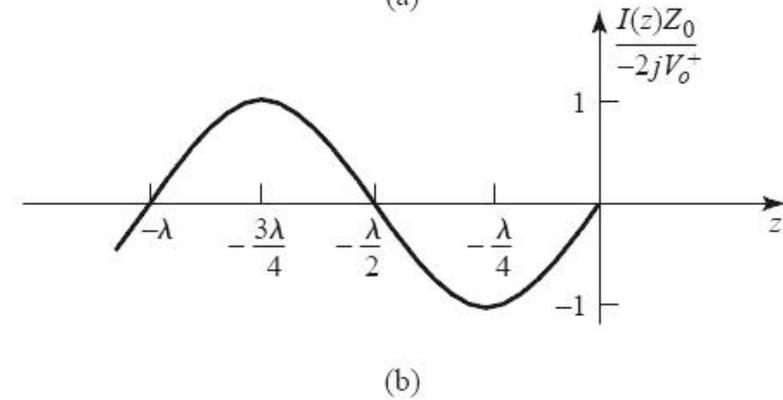
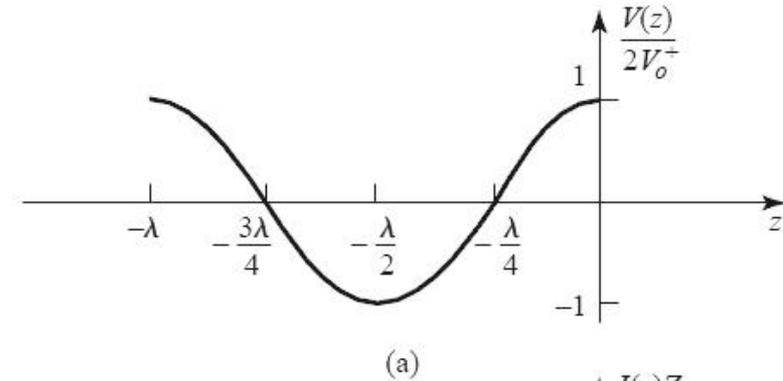
(c)

# Linie in gol

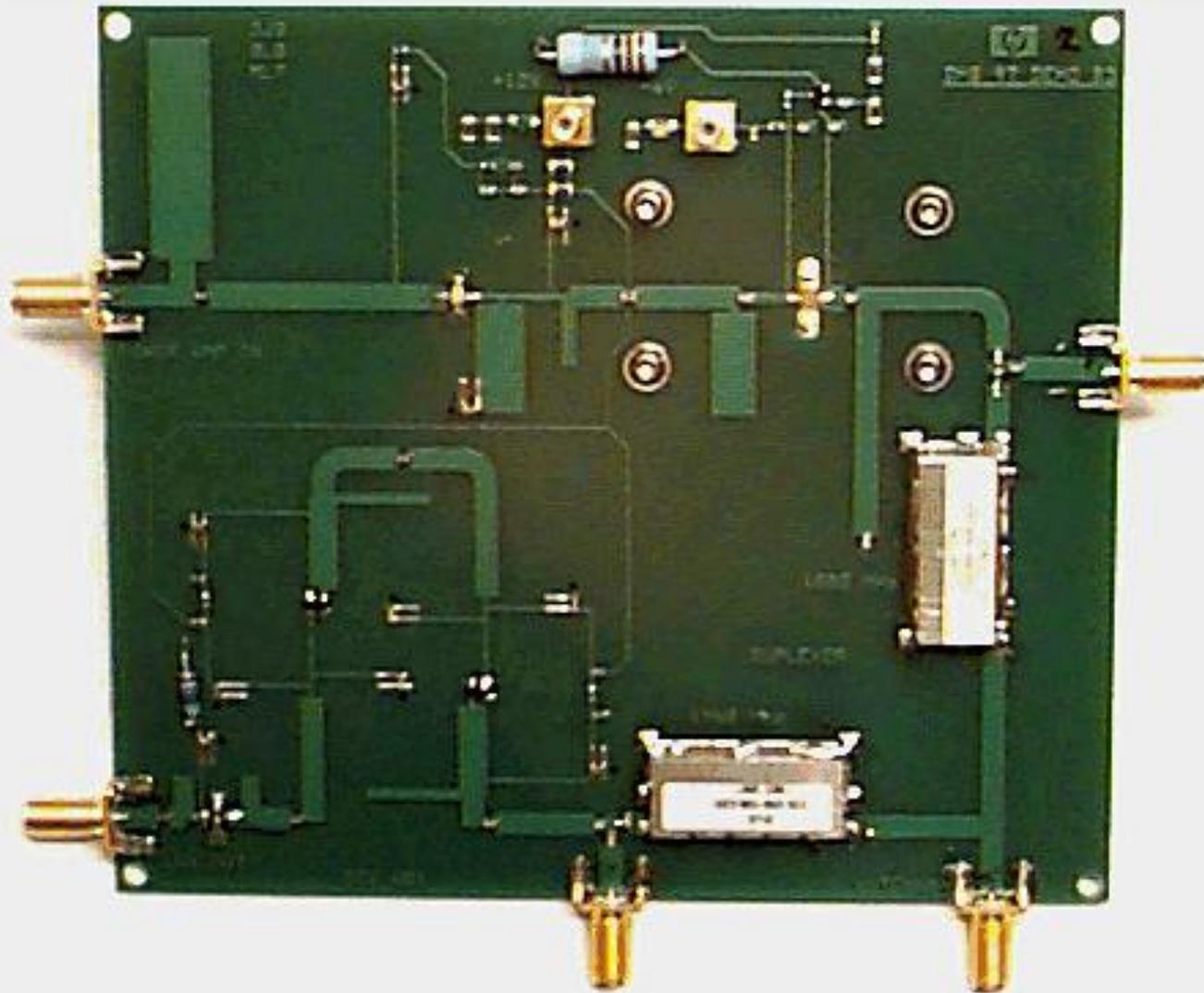
- $Z_L = \infty \rightarrow 1/Z_L = 0$
- reactanța pură
  - +/-  $\rightarrow$  în funcție de  $l$

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

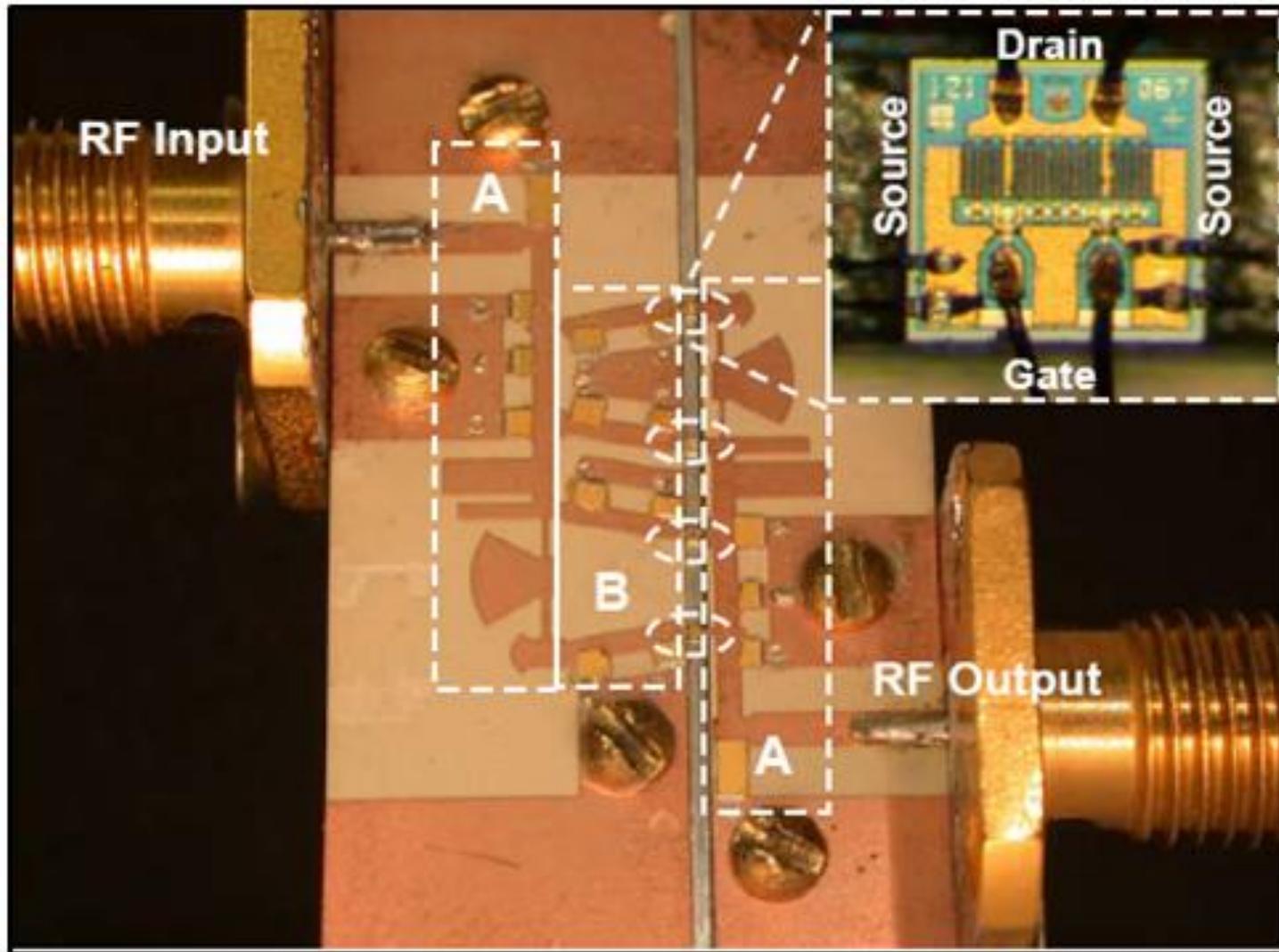
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



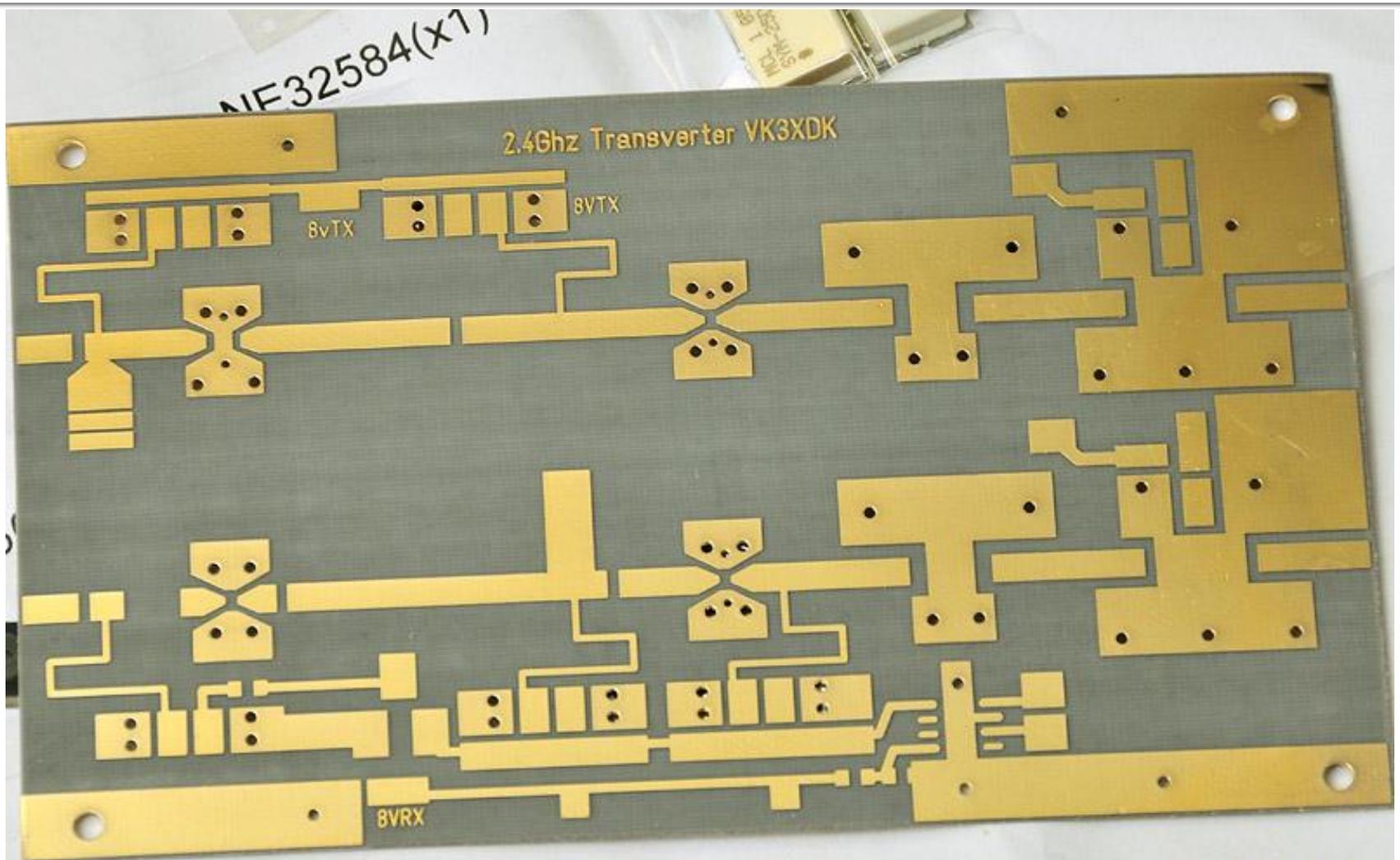
# Exemple



# Example

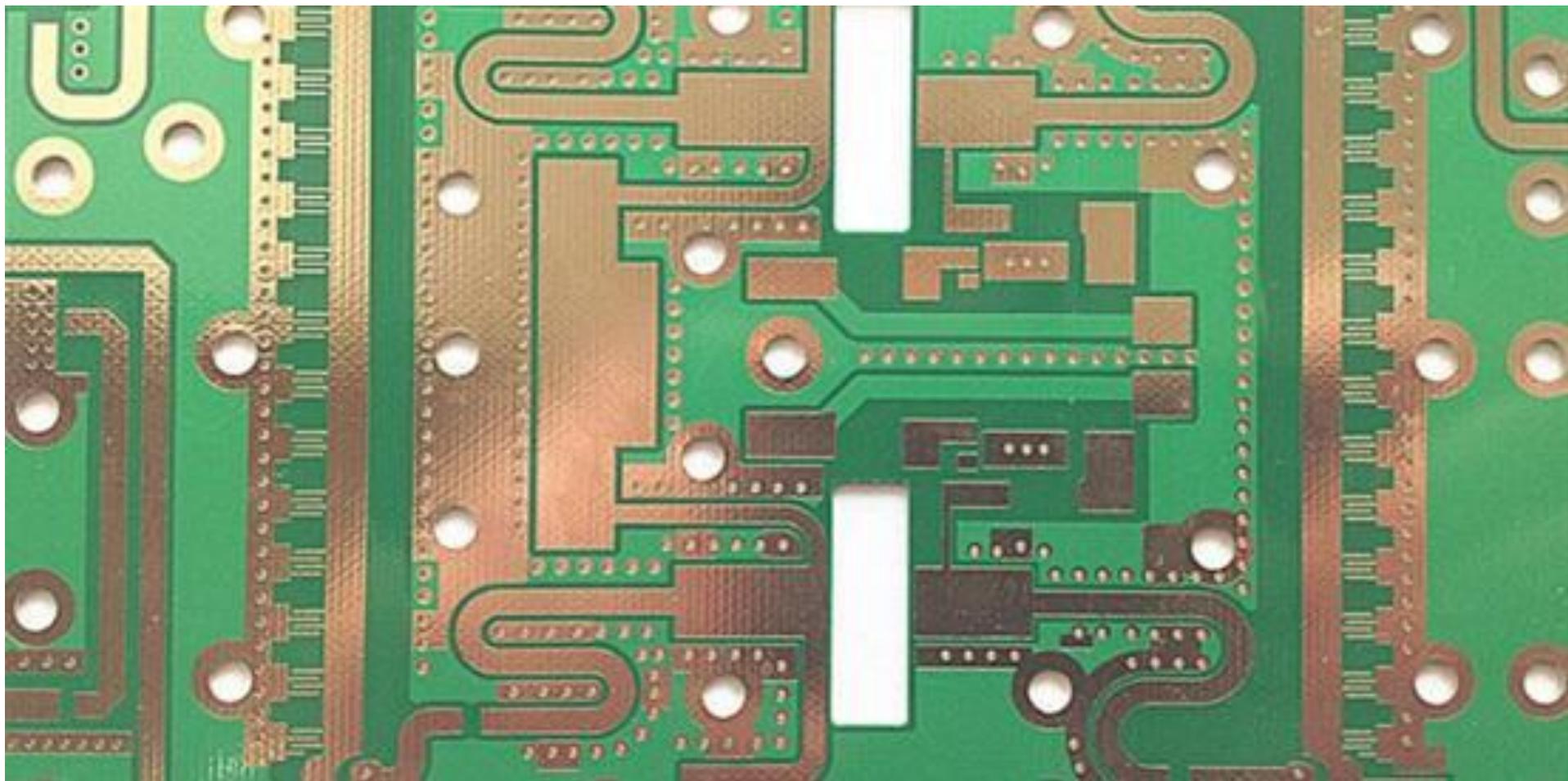


# Exemple

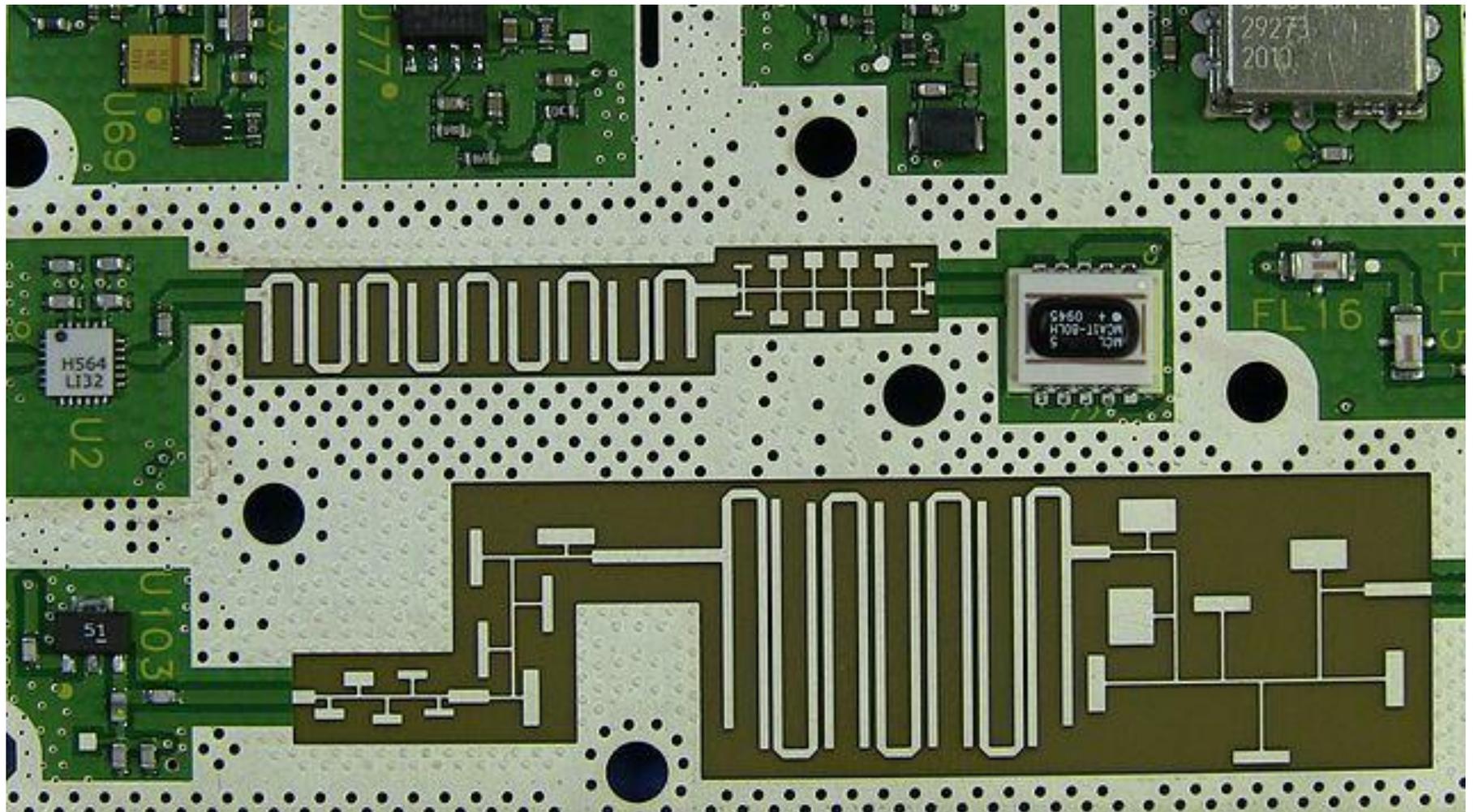


VK4CP

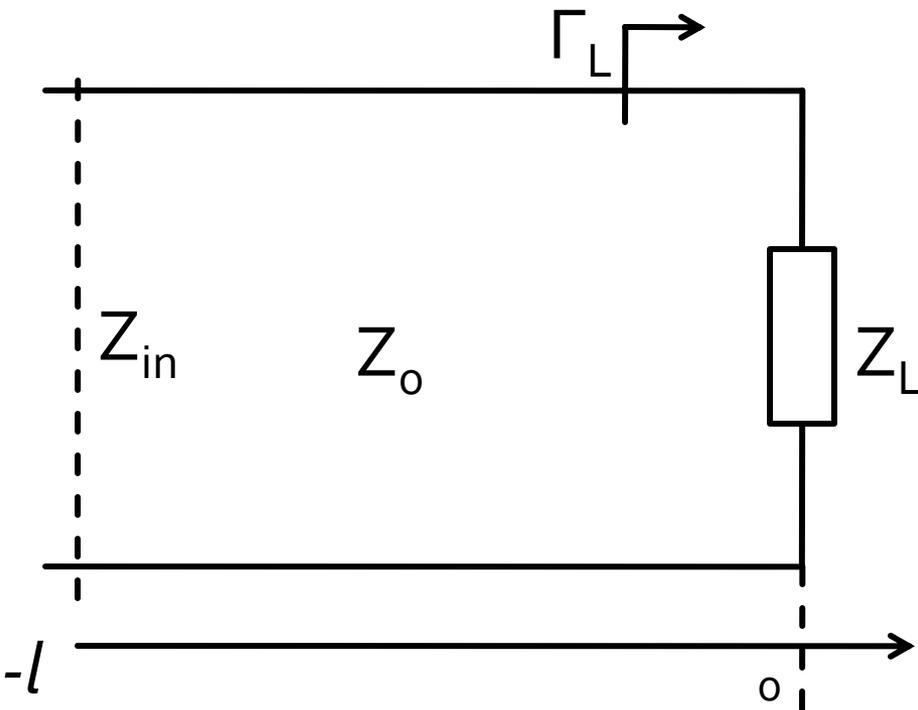
# Exemple



# Exemple



# Linie fara pierderi +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2j \cdot \beta \cdot l}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}$$

# Factor de unda stationara

$$V(z) = V_0^+ \cdot (e^{-j\beta \cdot z} + \Gamma \cdot e^{j\beta \cdot z}) \quad |V(z)| = |V_0^+| \cdot |e^{-j\beta \cdot z}| \cdot |1 + \Gamma \cdot e^{2j\beta \cdot z}| \quad \Gamma = |\Gamma| \cdot e^{j\theta}$$

$$|V(z)| = |V_0^+| \cdot |1 + |\Gamma| \cdot e^{\theta + 2j\beta \cdot z}|$$

amplitudine maxima pentru  $e^{\theta + 2j\beta \cdot z} = 1$   $V_{\max} = |V_0^+| \cdot (1 + |\Gamma|)$

amplitudine minima pentru  $e^{\theta + 2j\beta \cdot z} = -1$   $V_{\min} = |V_0^+| \cdot (1 - |\Gamma|)$

- se definește factorul de unda stationara

- (Voltage) Standing Wave Ratio

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

- număr real  $1 \leq VSWR < \infty$

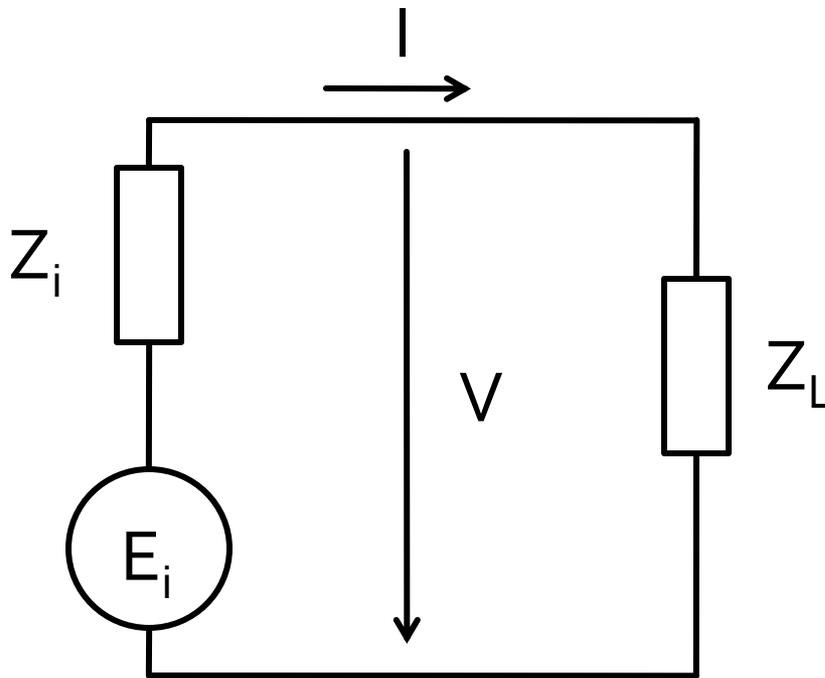
- o măsură a dezadaptării (SWR = 1 semnifică adaptare)

Transfer de putere

# Adaptarea de impedanță

# Adaptare

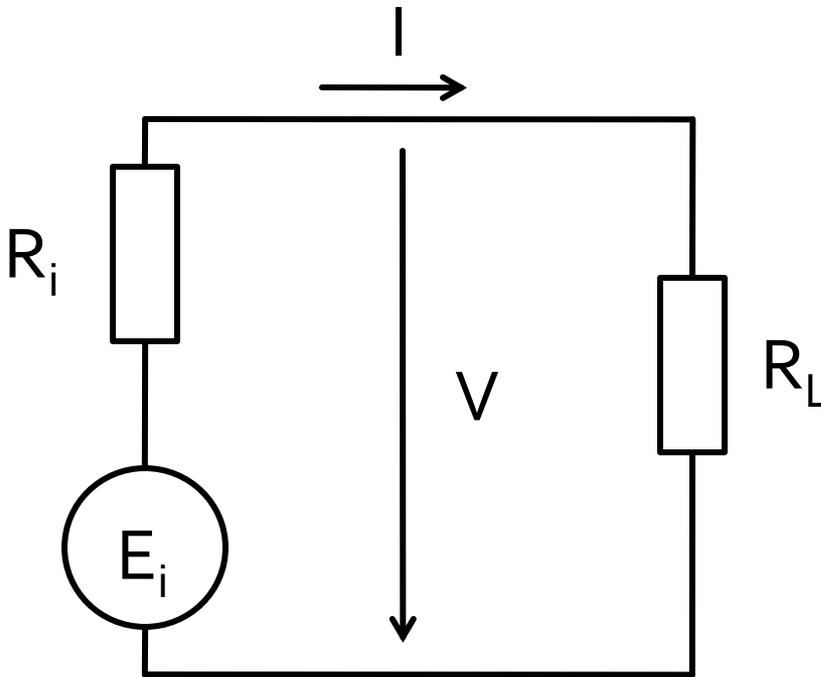
- Generator adaptat la sarcina ?



- valori impedanta ?
- reflexii ?

# Adaptare, impedante reale

- Generator adaptat la sarcina



$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

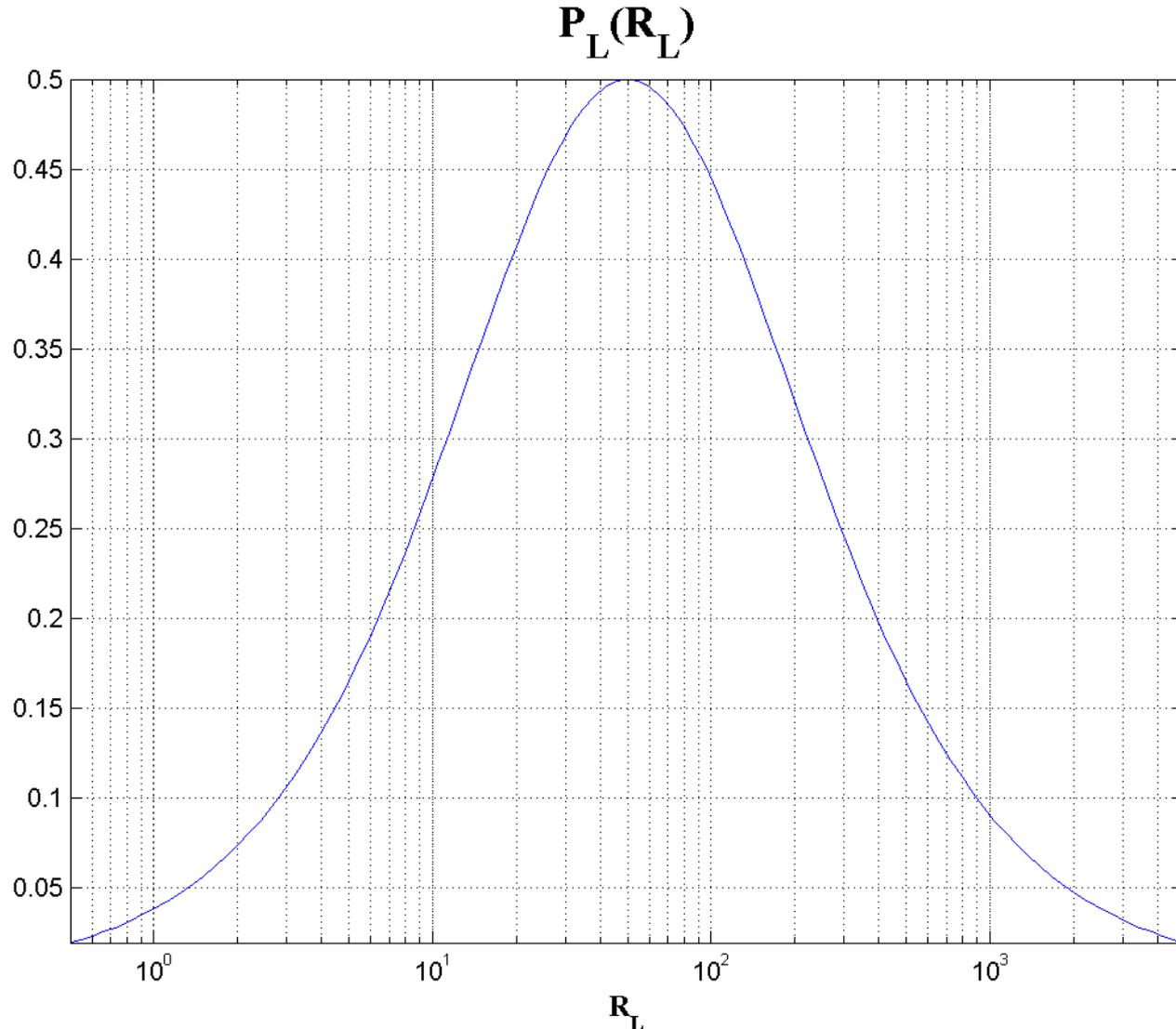
$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

# Adaptare , impedante reale

$$P_L = R_L \cdot I^2 \quad P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

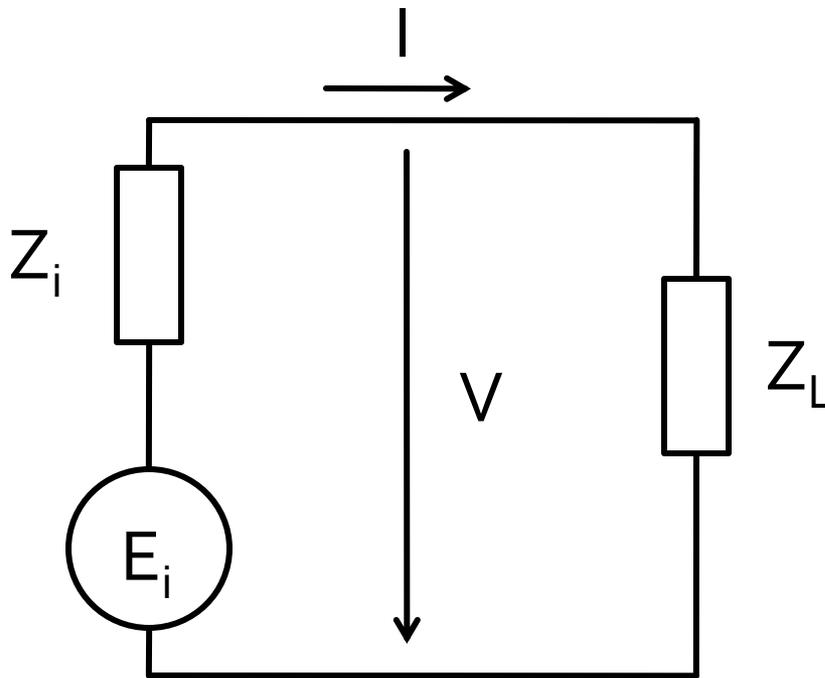
- Putere pe sarcina
  - $R_i = 50\Omega$
  - $R_L = 0 \rightarrow P_L = 0$
  - $R_L = \infty \rightarrow P_L = 0$

# Adaptare , impedante reale



# Adaptare, impedante complexe

- Generator adaptat la sarcina



$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \operatorname{Re} Z_L \cdot |I|^2$$

$$P_L = \operatorname{Re} Z_L \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

# Adaptare

$$P_L = \frac{R_L \cdot |E_i|^2}{|Z_i + Z_L|^2} = \frac{R_L \cdot |E_i|^2}{|(R_i + R_L) + j \cdot (X_i + X_L)|^2}$$

$$|a + j \cdot b| = \sqrt{a^2 + b^2}$$

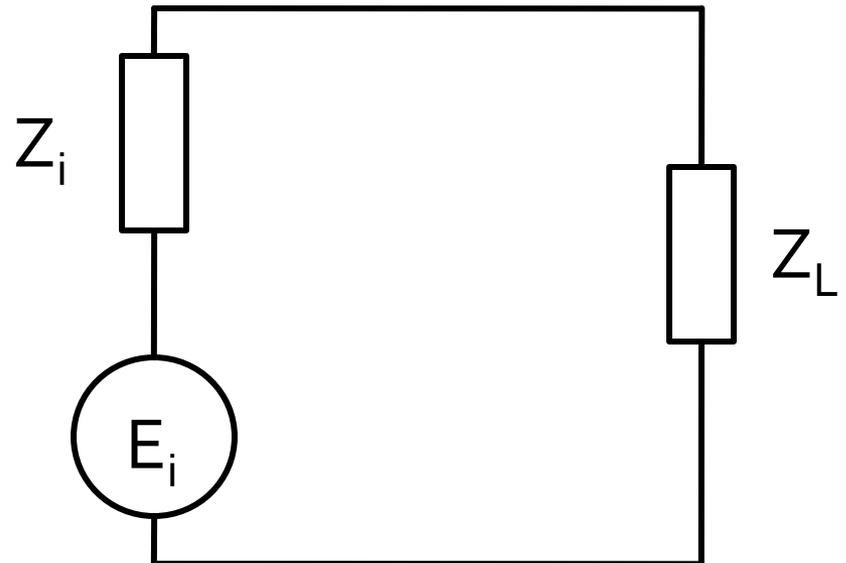
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

- Adaptare
  - putere maxima transmisa sarcinii
  - conditie?

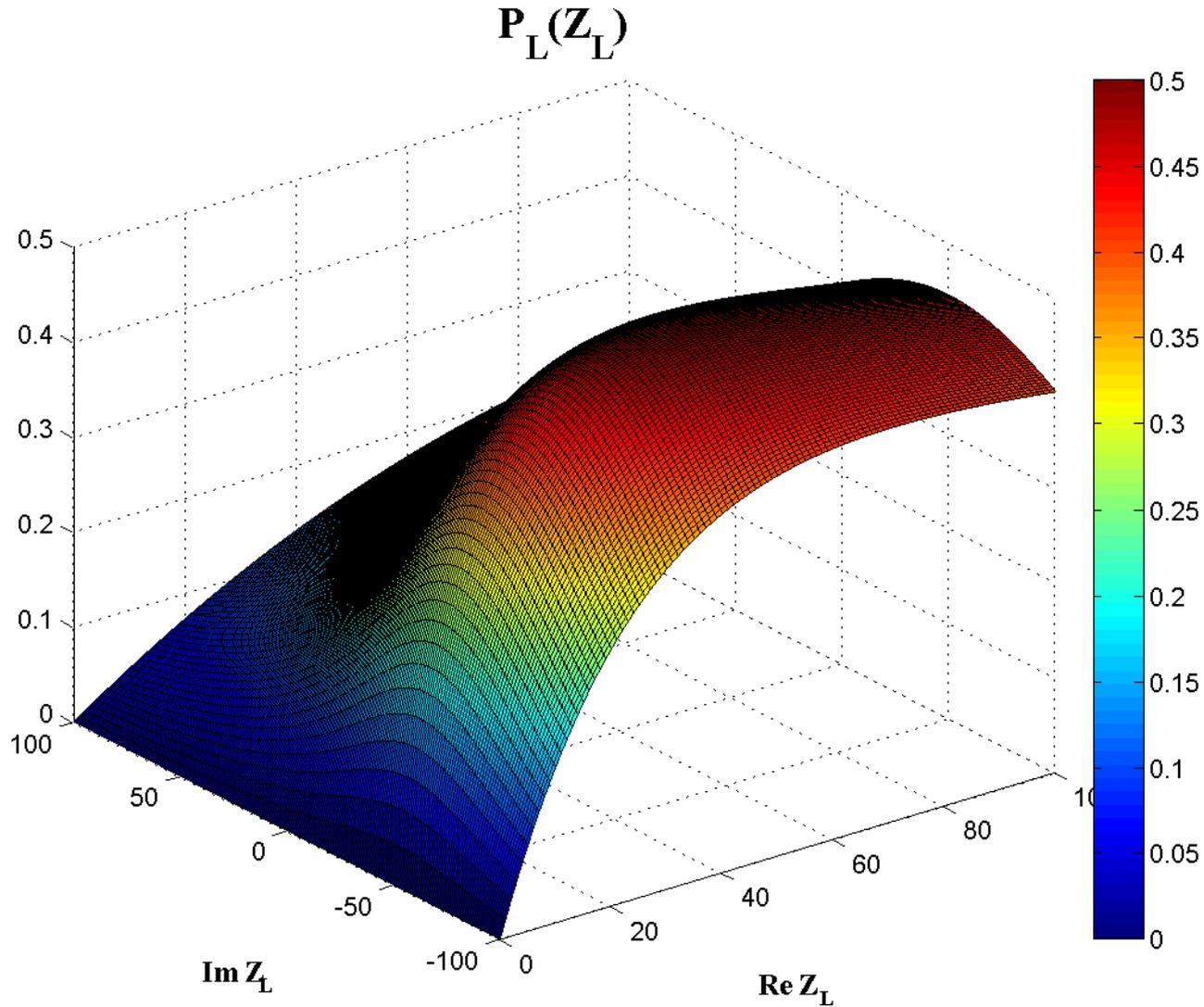
# Adaptare, exemplu

- $E = 10\text{V}$
- $Z_i = 50\ \Omega + j \cdot 50\ \Omega$
- $P_L(Z_L)$  ?

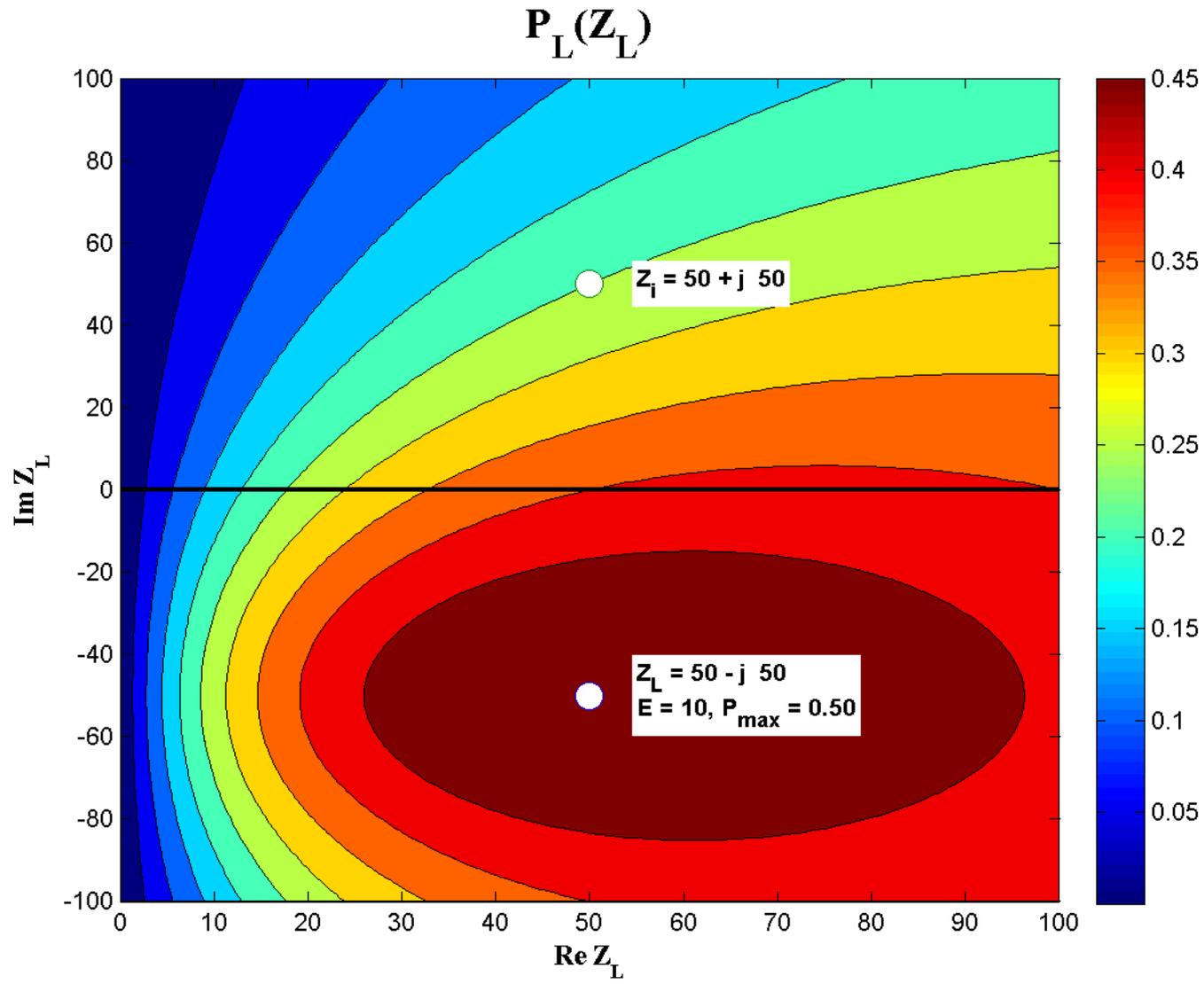
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$



# Adaptare, exemplu



# Adaptare, exemplu



# Adaptare dpdv al puterii

$$R_i > 0, R_L > 0 \quad P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

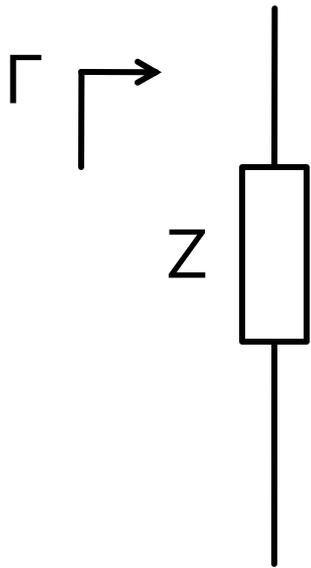
$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a \quad R_L = R_i, X_L = -X_i$$

- $P_a$  : Puterea disponibilă (available)

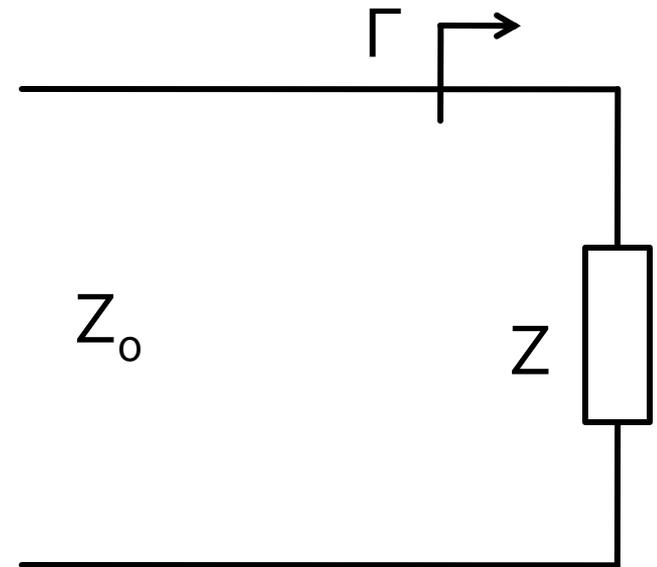
$$Z_L = Z_i^*$$

# Coeficient de reflexie

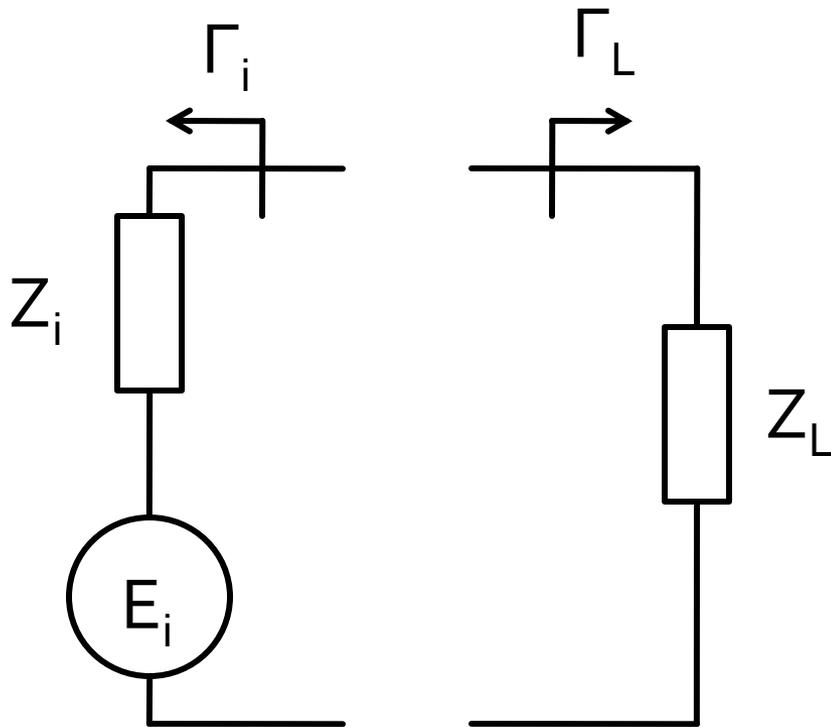
- Un  $Z_0$  oarecare ales ca referinta



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



# Adaptare dpdv al puterii



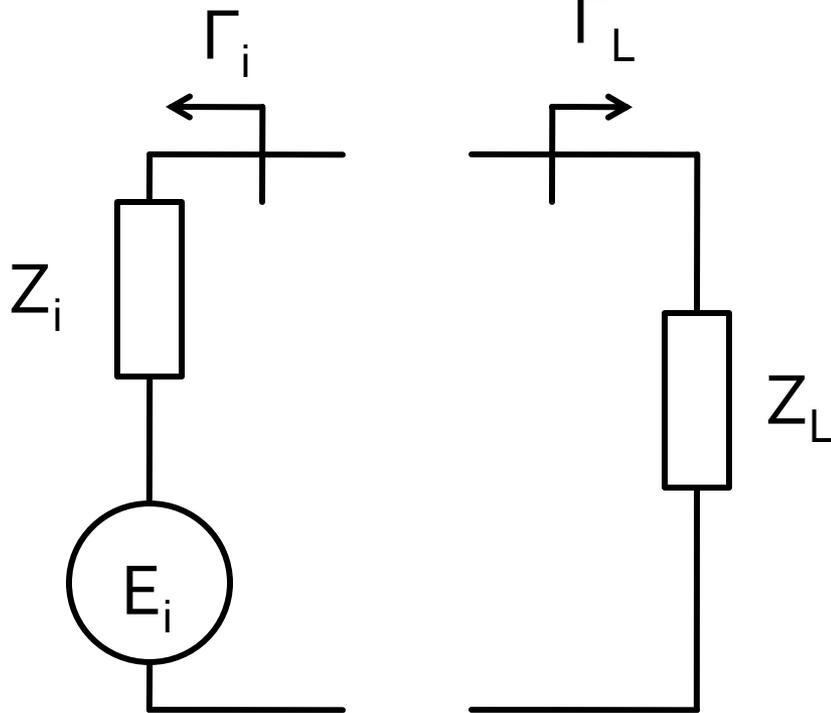
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

# Adaptare dpdv al puterii



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0^*} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

# Adaptare dpdv al puterii

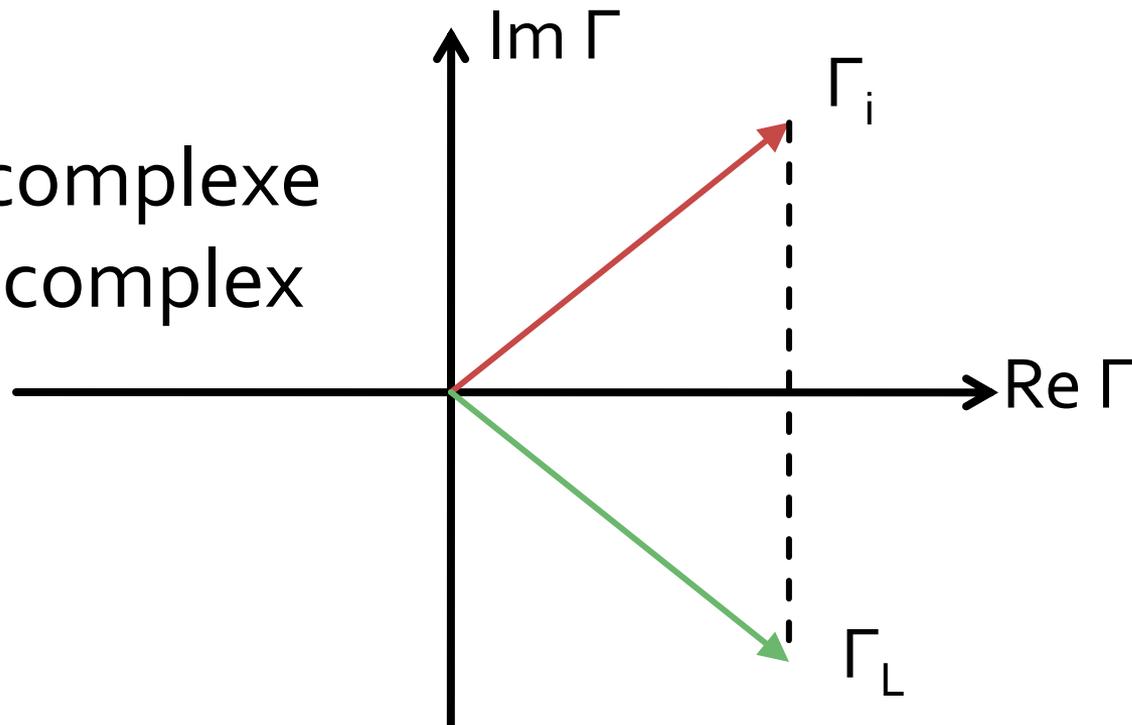
$$Z_L = Z_i^*$$

Daca se alege un  $Z_0$  real

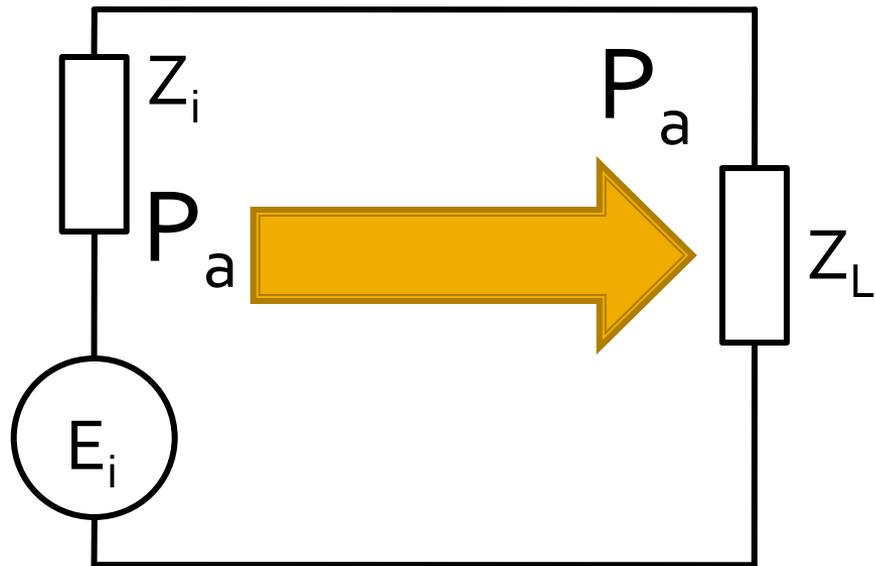
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- numere complexe
- in planul complex

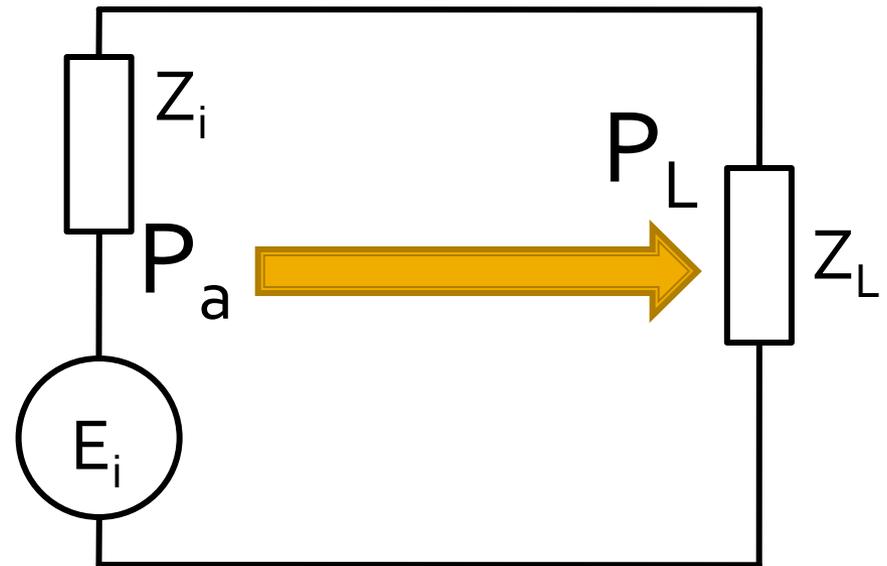


# Reflexie de putere / Model



$$Z_L = Z_i^*$$

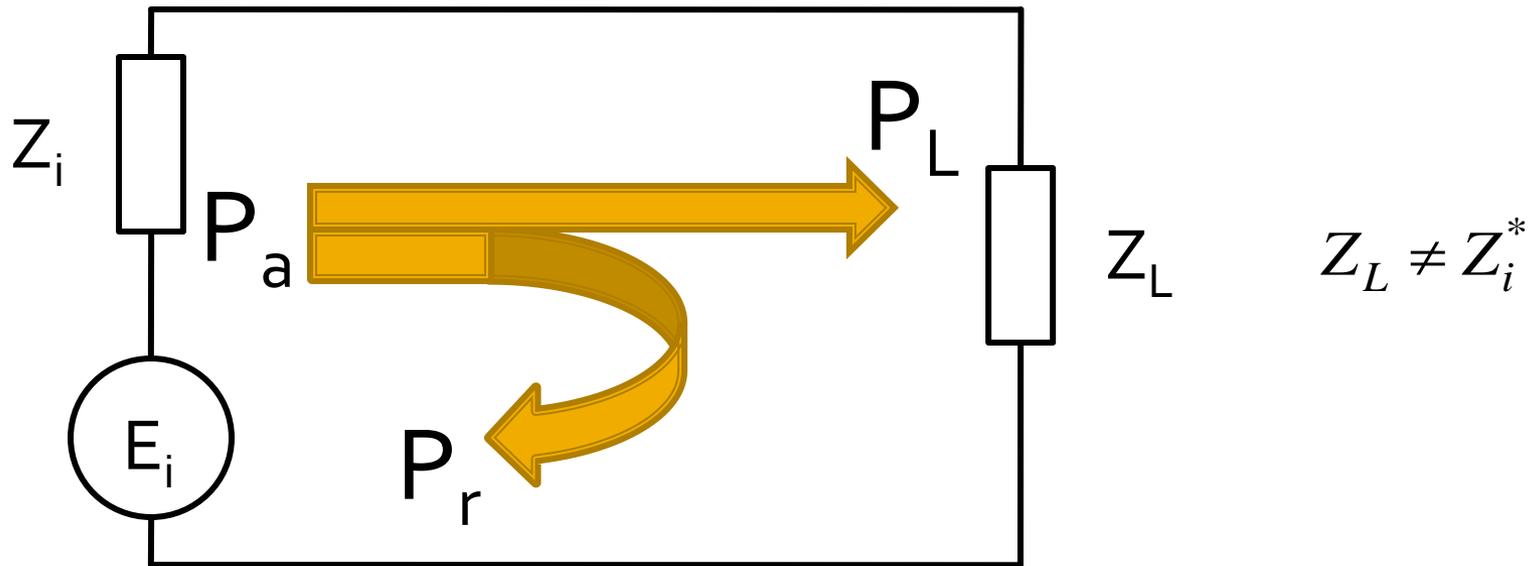
$$P_L = P_a$$



$$Z_L \neq Z_i^*$$

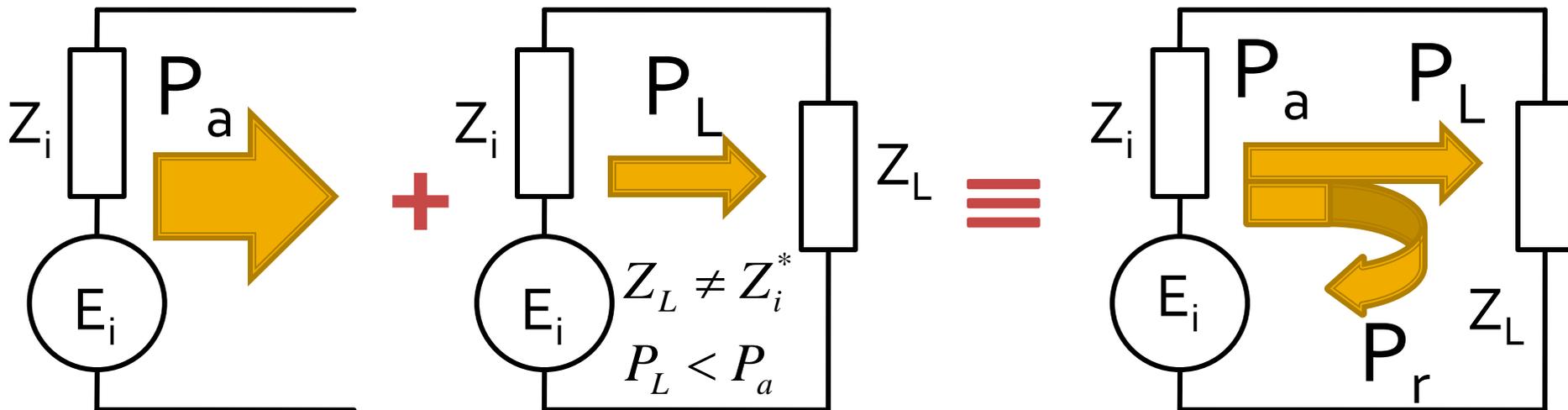
$$P_L < P_a$$

# Reflexie de putere / Model



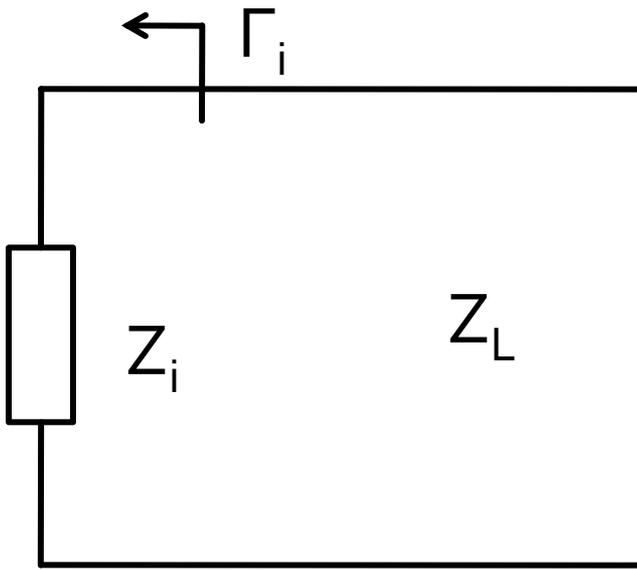
- ~~Putere reflectata~~
- Putere a undei reflectate

# Reflexie de putere / Model

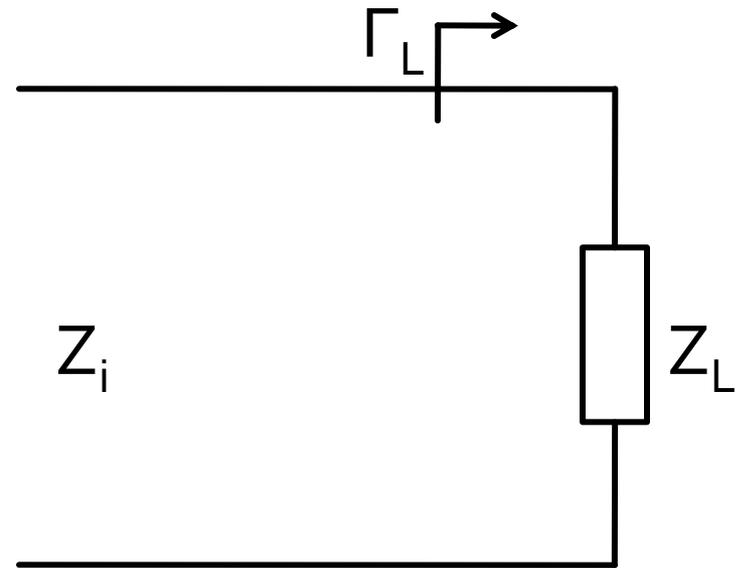


- Generatorul are posibilitatea de a oferi o anumita putere maxima de semnal  $P_a$
- Pentru o sarcina oarecare, acestuia i se ofera o putere de semnal mai mica  $P_L < P_a$
- Se intampla "**ca si cum**" (model) o parte din putere se reflecta  $P_r = P_a - P_L$
- Puterea este o marime **scalara!**

# Coeficienti de reflexie



$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

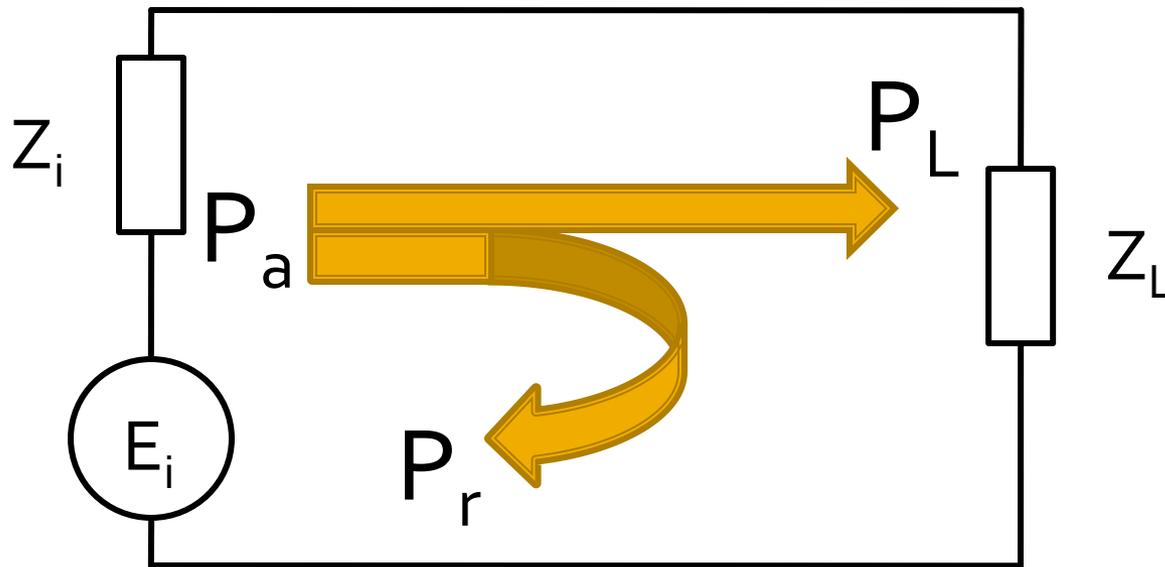
# Coeficienti de reflexie

$$\Gamma_i = \frac{(R_i - R_L) + j \cdot (X_i + X_L)}{(R_i + R_L) + j \cdot (X_i + X_L)} \quad \Gamma_L = \frac{(R_L - R_i) + j \cdot (X_L + X_i)}{(R_L + R_i) + j \cdot (X_L + X_i)}$$

$$|\Gamma_i| = \frac{|(R_i - R_L) + j \cdot (X_i + X_L)|}{|(R_i + R_L) + j \cdot (X_i + X_L)|} = \frac{\sqrt{(R_i - R_L)^2 + (X_i + X_L)^2}}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} = |\Gamma_L|$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

# Reflexie de putere / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} - \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2} = \frac{|E_i|^2}{4R_i} \cdot \left[ 1 - \frac{4R_L \cdot R_i}{(R_i + R_L)^2 + (X_i + X_L)^2} \right]$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[ \frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

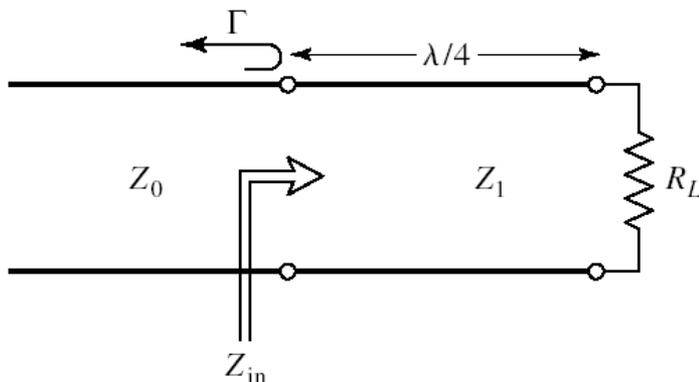
- $|\Gamma|^2$  **este** un coeficient de reflexie in putere

Transformatorul in sfert de lungime de unda

# **Adaptarea de impedanța**

# Transformatorul in sfert de lungime de unda

- Feed line – linie de intrare cu impedanta caracteristica  $Z_0$
- Sarcina cu impedanta (**rezistiva!**)  $R_L$
- Dorim adaptarea sarcinei la fider cu o linie de lungime  $\lambda/4$  si impedanta caracteristica  $Z_1$

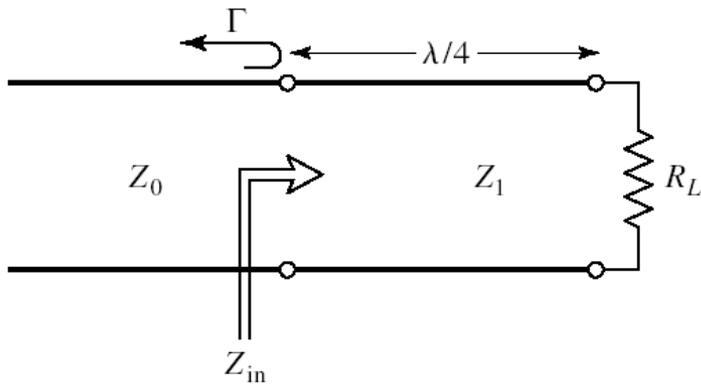


$$Z_{in} = Z_1 \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}}$$

$$\Gamma_o = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

# Transformatorul in sfert de lungime de unda



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L}$$

$$\Gamma_{in} = 0 \Rightarrow Z_1 = \sqrt{Z_0 R_L}$$

- Pe fider ( $Z_0$ ) avem doar unda progresiva
- Pe linia in sfert de lungime de unda ( $Z_1$ ) avem unda stationara

# Transformatorul in sfert de lungime de unda

## ■ Punct de vedere fizic

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \dots$$

$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

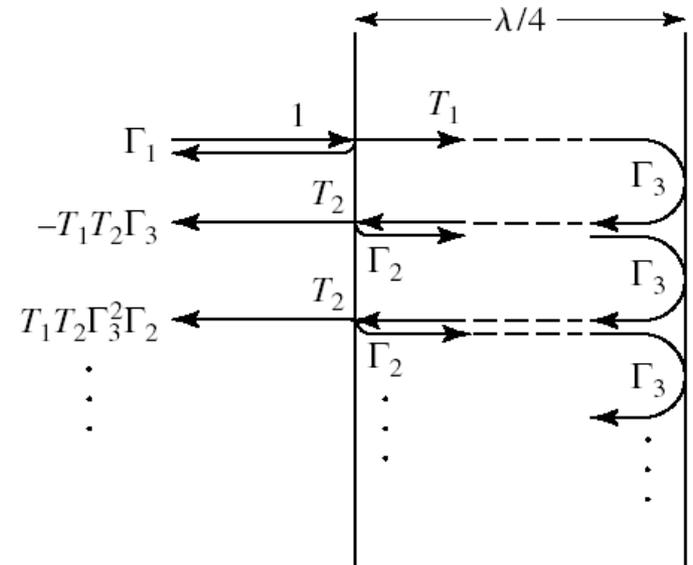
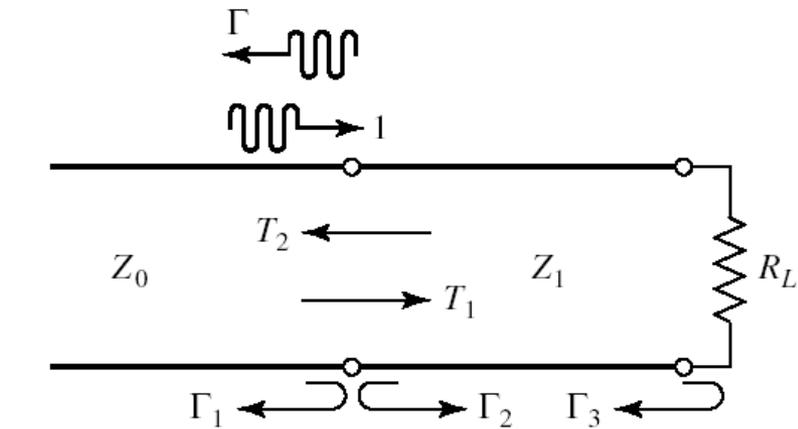
$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$

$$\left. \begin{array}{l} T_1 \\ T_2 \end{array} \right\} T = 1 - \Gamma$$



# Transformatorul in sfert de lungime de unda

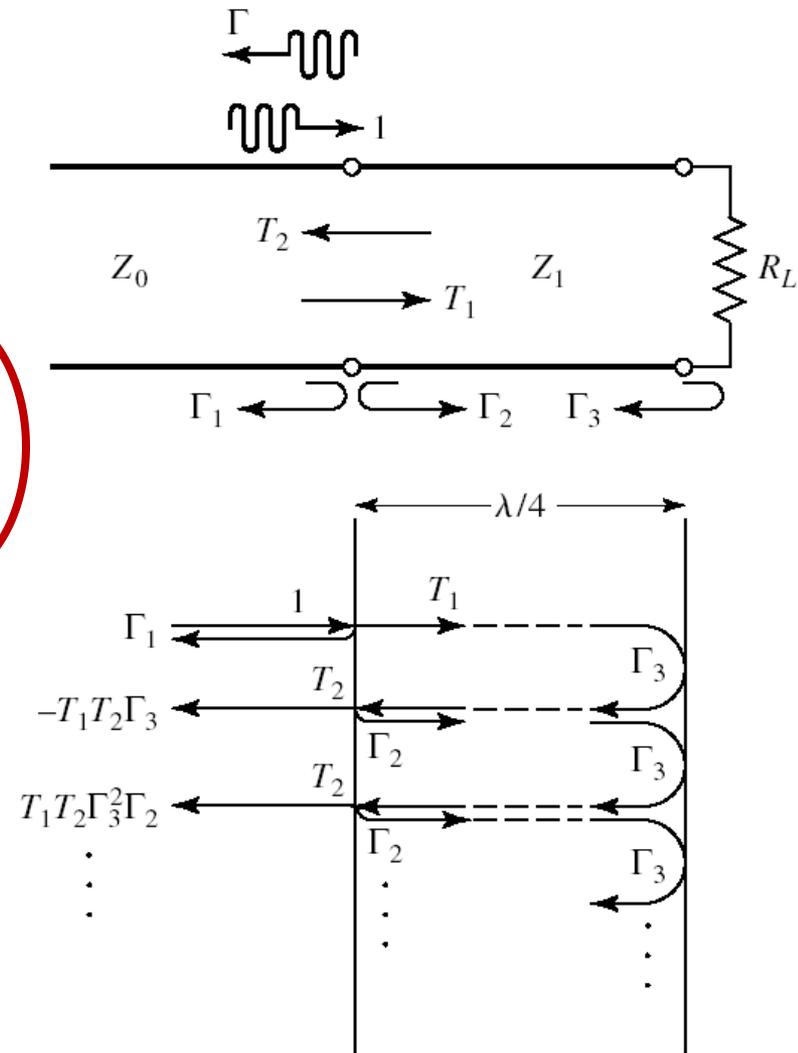
## ■ Punct de vedere fizic

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad \text{for } |x| < 1,$$

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$

$$\Gamma_1 - \Gamma_3(\Gamma_1^2 + T_1 T_2) = \frac{2(Z_1^2 - Z_0 R_L)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma = 0 \Leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$



# Característica de frecuencia

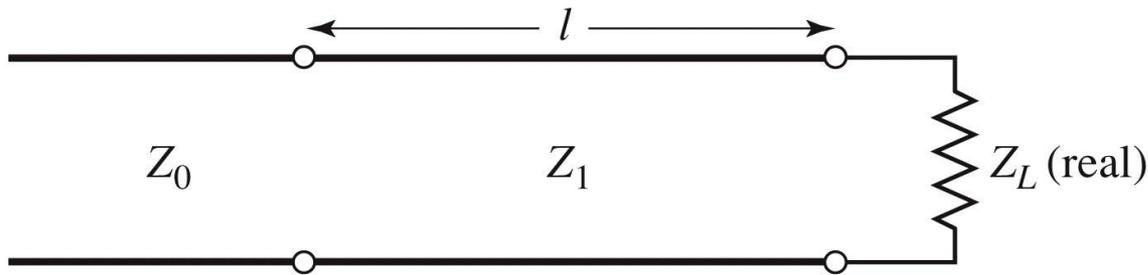


Figure 5.10  
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$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (doar) la frecventa  $f_0$

$$l = \frac{\lambda_0}{4} \quad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$\theta = \beta \cdot l$$

$$t = \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt(Z_1^2 - Z_0 Z_L)}{Z_1(Z_L + Z_0) + jt(Z_1^2 + Z_0 Z_L)}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}$$

# Caracteristica de frecventa

- calitatea adaptarii  $\equiv$  coeficient de reflexie in putere

$$\begin{aligned} |\Gamma| &= \frac{|Z_L - Z_0|}{[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L]^{1/2}} \\ &= \frac{1}{\{(Z_L + Z_0)^2 / (Z_L - Z_0)^2 + [4t^2 Z_0 Z_L / (Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L / (Z_L - Z_0)^2] + [4Z_0 Z_L t^2 / (Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L / (Z_L - Z_0)^2] \sec^2 \theta\}^{1/2}}, \end{aligned}$$

$$\sec \theta = \frac{1}{\cos \theta} \rightarrow$$

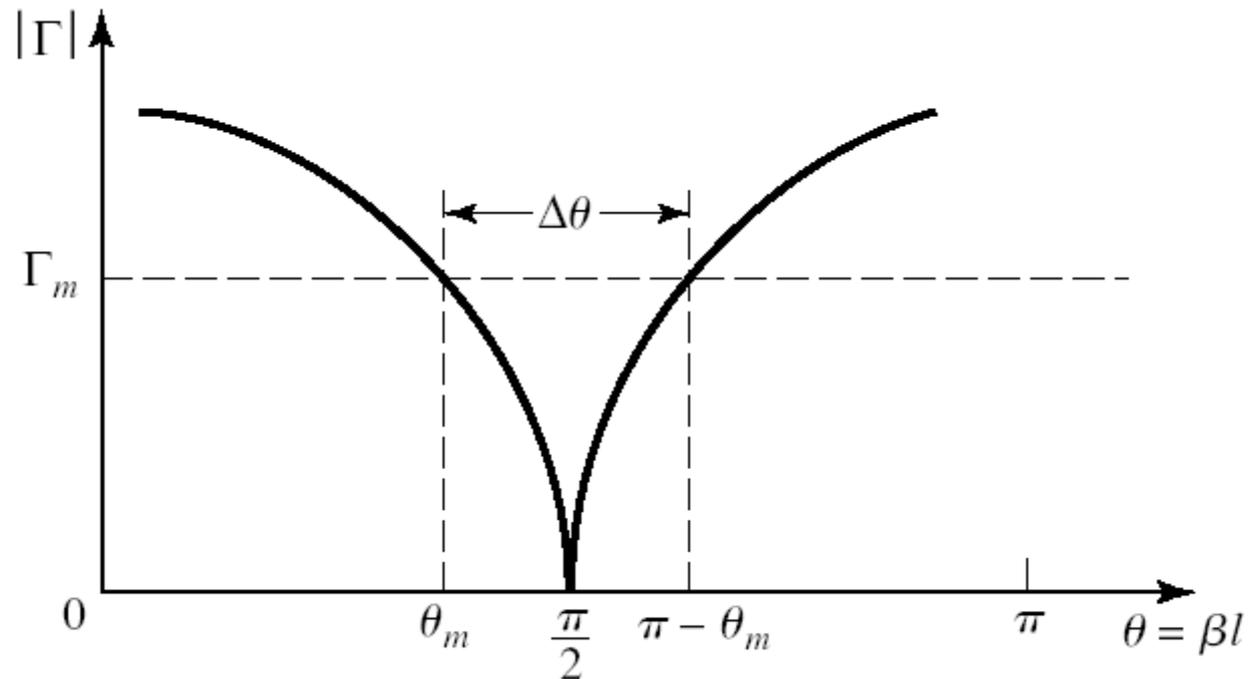
$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + t^2$$

# Caracteristica de frecventa

- ne intereseaza frecventa in jurul frecventei la care facem adaptarea (banda ingusta)

$$f \approx f_0 \quad l \approx \frac{\lambda_0}{4} \quad \theta \approx \frac{\pi}{2} \quad \sec^2 \theta = 1 + \tan^2 \theta \gg 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos \theta|$$



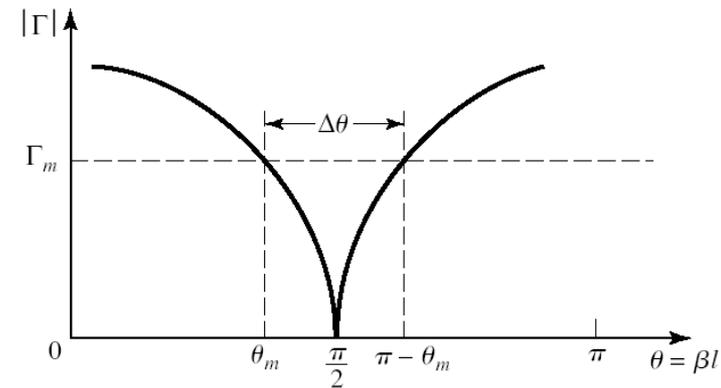
# Caracteristica de frecventa

- Definim un maxim acceptat pentru coeficientul de reflexie  $\Gamma_m$  care va defini banda adaptarii,  $\theta_m$

$$\frac{1}{\Gamma_m^2} = 1 + \left( \frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0} \sec \theta_m \right)^2,$$

$$\cos \theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2 \left( \frac{\pi}{2} - \theta_m \right)$$



- in linii TEM

$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2f_0}$$

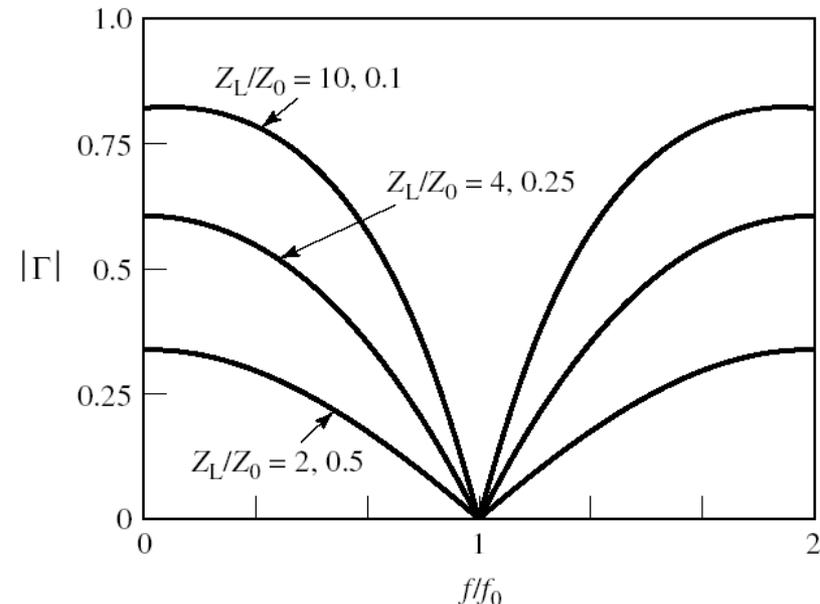
$$f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|} \right]$$

# Caracteristica de frecventa

- Pentru linii non TEM constanta de propagare nu depinde linear de frecventa, dar in practica influenta este minora in banda ingusta
- Sunt neglijate reactantele introduse de discontinuitati ( $Z_0 \rightarrow Z_1$ ). Compensarea se face printr-o mica modificare a lungimii liniei
- Banda depinde de dezadaptarea initiala

cu cat dezadaptarea este mai mica  
cu atat banda se obtine mai larga



# Exemplu

- Transformator de adaptare cu o singura sectiune ( $\lambda/4$ ) pentru a adapta o sarcina de  $10\Omega$  la o linie de  $50\Omega$  la frecventa  $f_0=3\text{GHz}$ 
  - banda pentru  $\text{SWR}<1.5$

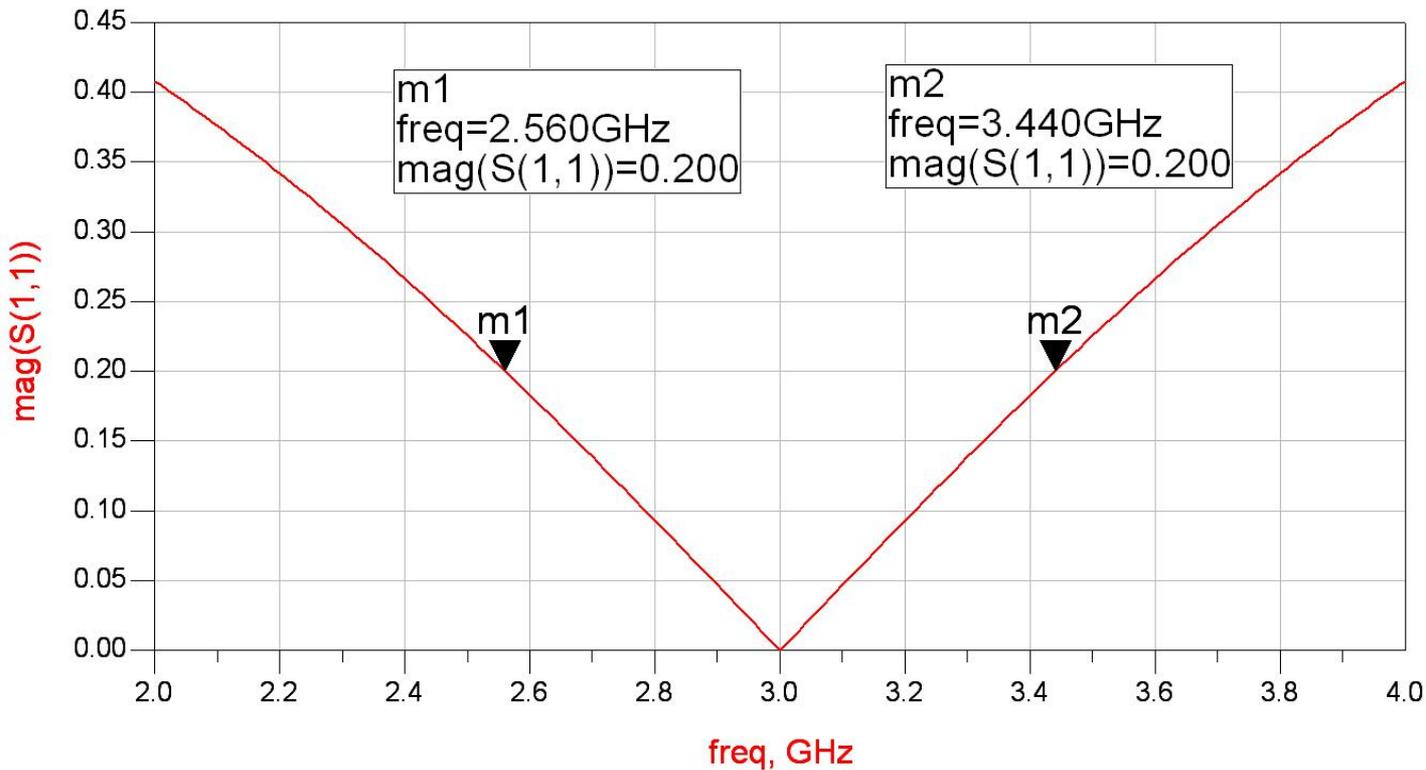
$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36 \Omega,$$

$$\Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

$$\begin{aligned} \frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|} \right] \\ &= 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|} \right] \\ &= 0.29, \text{ or } 29\%. \end{aligned}$$

# Simulare

## ■ simulare ADS

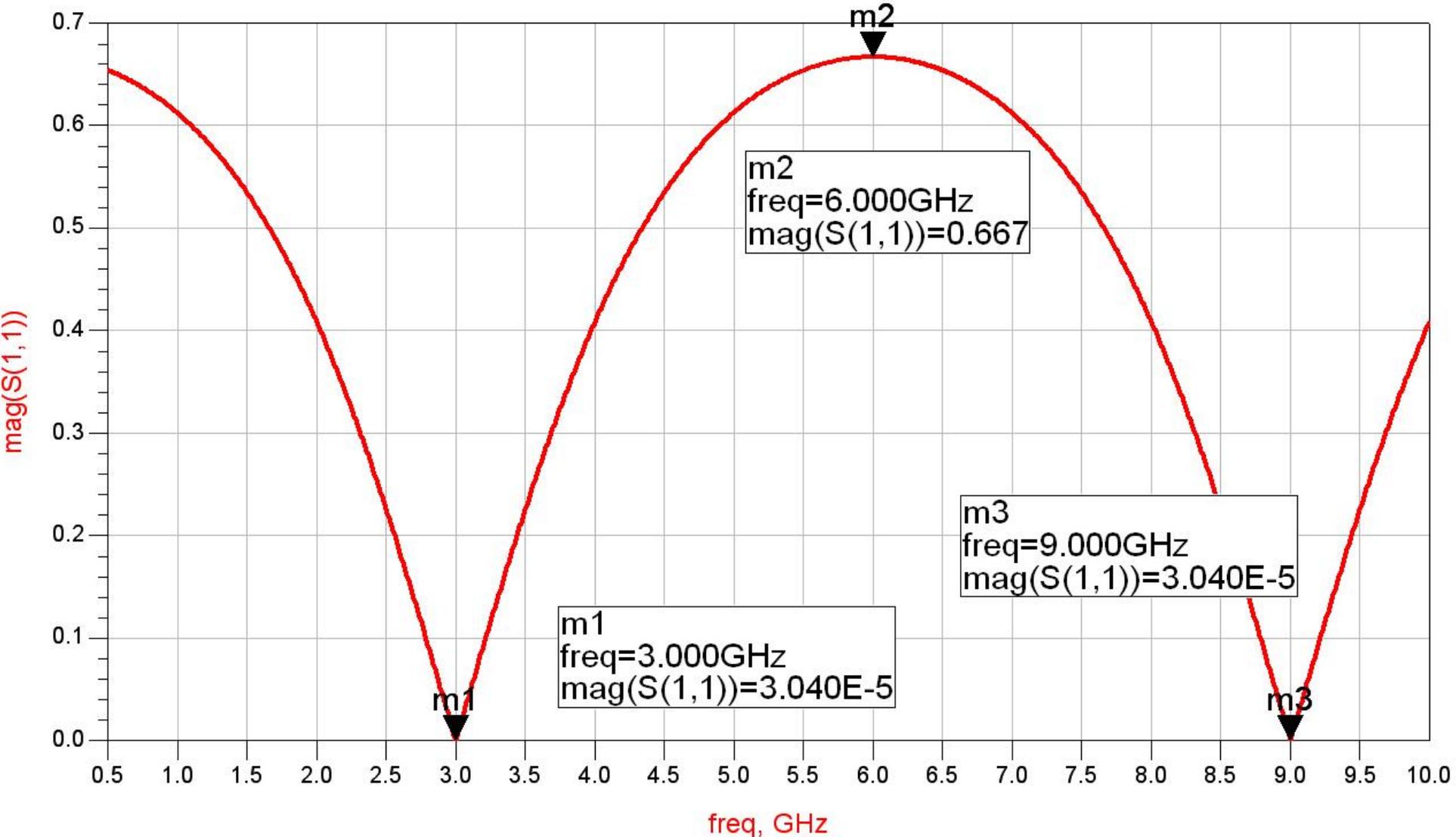


$$\Delta f = 0.88 \text{GHz}$$

$$|\Gamma(3\text{GHz})| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

# Simulare banda larga



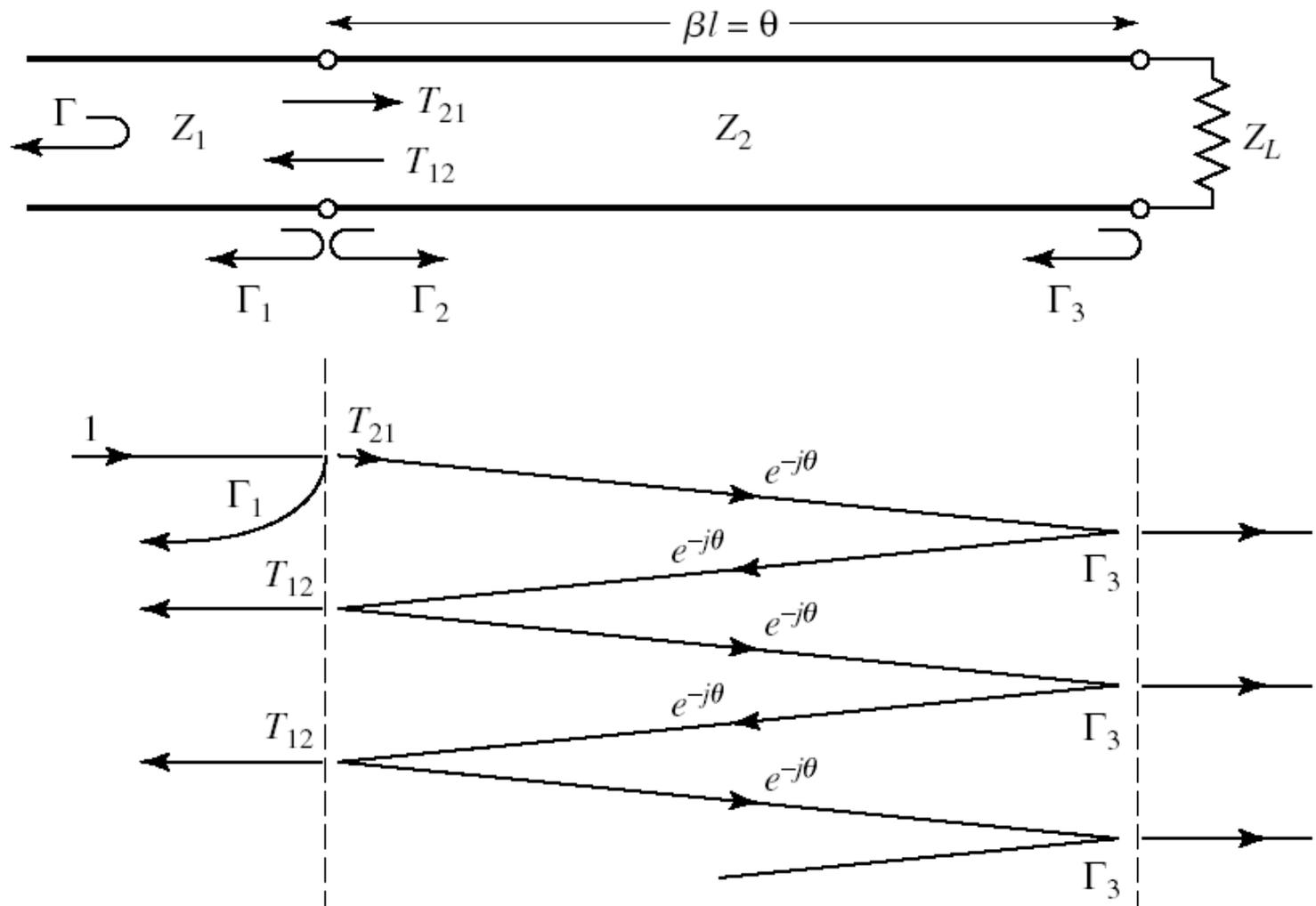
Adaptarea cu transformatoare de impedanta (Lab. 1)

# Adaptarea de impedanța

# Transformatoare de impedanta multisectiune

- Transformatorul in sfert de lungime de unda permite adaptarea oricarei impedante reale cu orice impedanta a fiderului (liniei).
- Daca banda necesara este mai mare decat cea oferita de transformatorul in sfert de lungime de unda se folosesc transformatoare multisectiune
  - caracteristica binomiala
  - tip Cebîșev

# Teoria reflexiilor mici



# Teoria reflexiilor mici

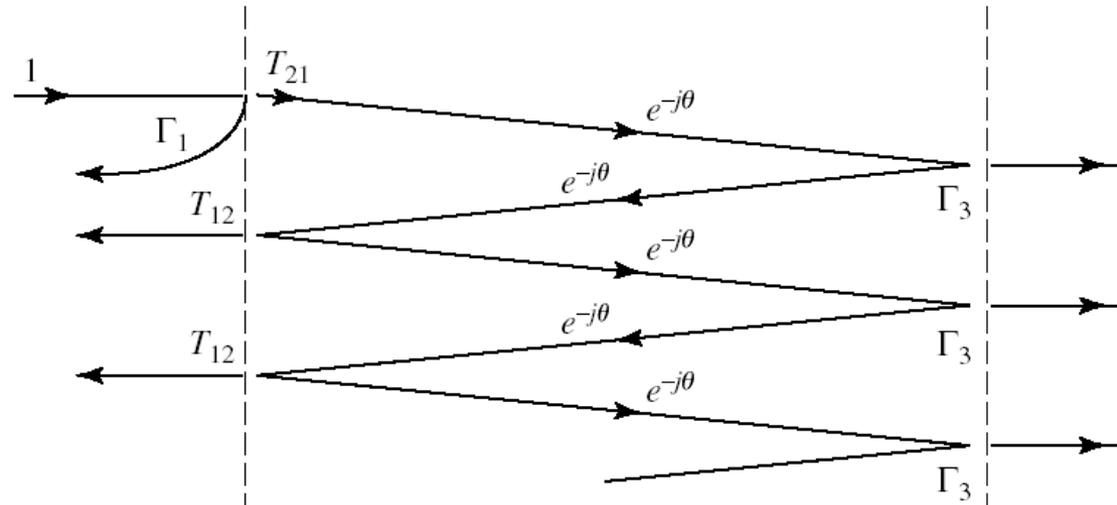
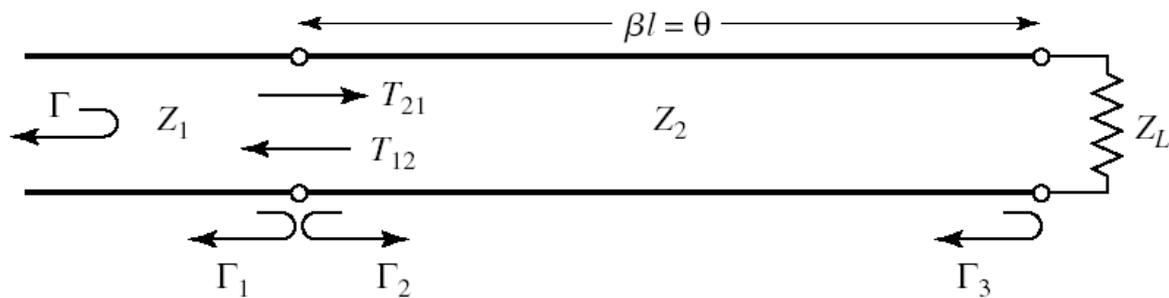
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$



$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \dots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

# Teoria reflexiilor mici

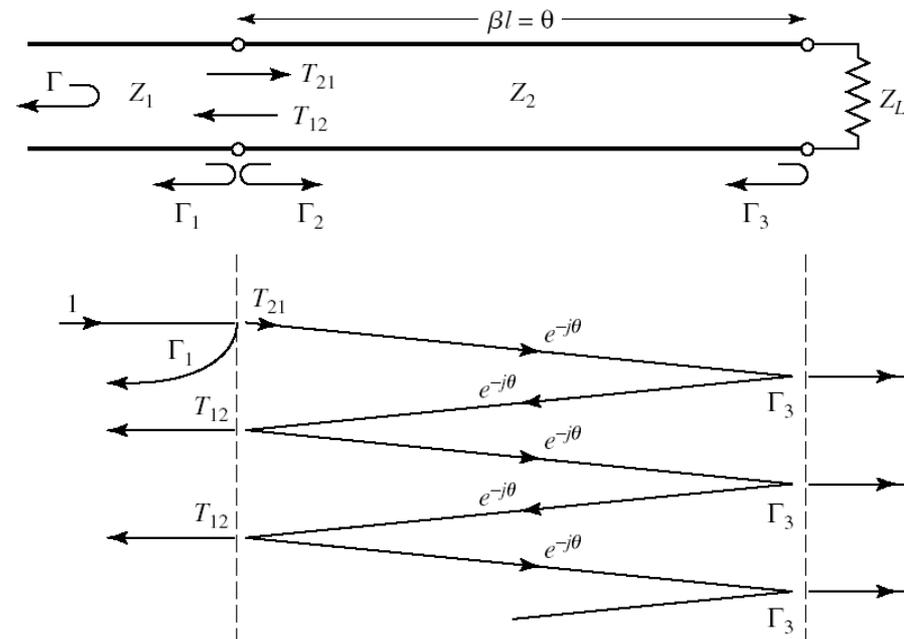
$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

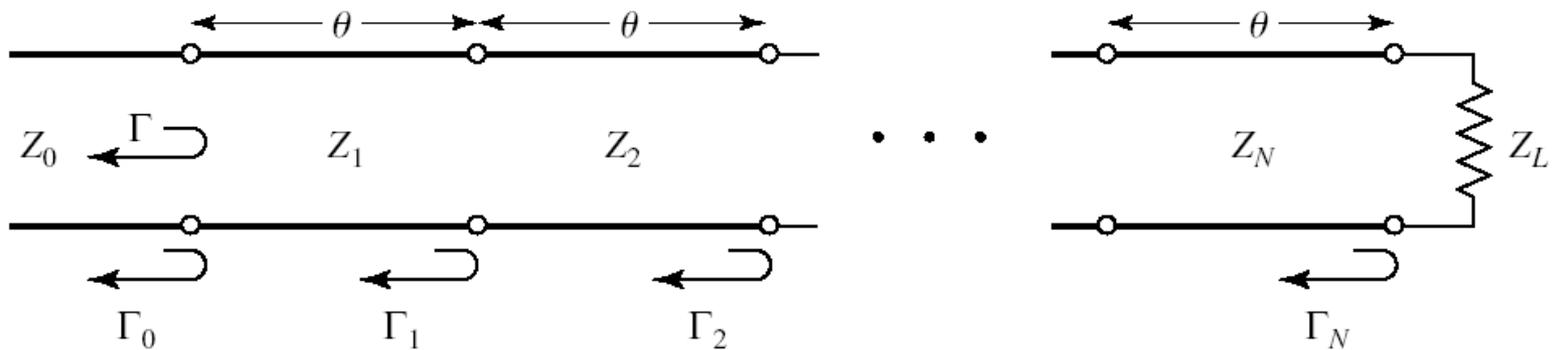
$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

- Dacă diferențele între  $Z_1 \div Z_2$  și  $Z_2 \div Z_L$  sunt mici putem aproxima:

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



# Transformatoare cu mai multe sectiuni



- Presupunem ca toate impedantele **cresc sau descrec uniform**

- Toti coeficientii de reflexie vor fi reali si de acelasi semn

- Anterior  $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$n = \overline{1, N-1}$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

# Transformatoare cu mai multe sectiuni

- Realizez transformatorul **simetric**

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \dots$$

- Aceasta **nu** implica faptul ca impedantele sunt egale

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot [\Gamma_0 \cdot (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 \cdot (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \Gamma_2 \cdot (e^{j(N-4)\theta} + e^{-j(N-4)\theta}) + \dots]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:

$$\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$

# Transformatoare cu mai multe sectiuni

- Coeficient de reflexie

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \dots + a_N \cdot x^N$$

- aleg coeficientii astfel incat sa obtin o variatie dorita (a polinomului)

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Raspunsul acestui transformator este de tip maxim plat in jurul frecventei de adaptare
- Pentru N sectiuni se anuleaza primele N-1 derivate ale functiei  $|\Gamma(\theta)|$

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N$$

$$|\Gamma(\theta)| = |A| \cdot |e^{-j\theta}|^N \cdot |e^{j\theta} + e^{-j\theta}|^N = 2^N \cdot |A| \cdot |\cos \theta|^N$$

$$\left| \Gamma\left(\frac{\pi}{2}\right) \right| = 0; \quad \frac{d^n}{d\theta^n} |\Gamma(\theta)| \Big|_{\theta=\frac{\pi}{2}} = 0 \quad n = \overline{1, N-1} \quad l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2}$$

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- $A, \theta \rightarrow 0$ , liniile de lungime 0, dispar

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0} \quad A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- dezvoltarea binomului

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \dots + C_N^n \cdot x^n + \dots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!n!}$$

- Coeficientii de reflexie

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N \quad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

## ■ Proiectare

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \quad \Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n} \quad \ln x \cong 2 \cdot \frac{x-1}{x+1} \quad x \cong 1$$

$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

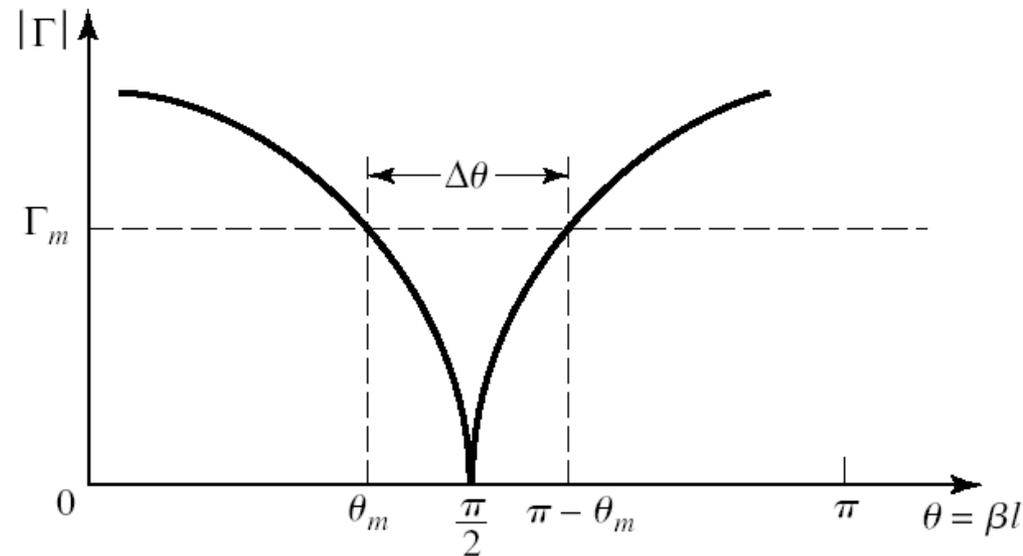
$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

# Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Banda,  $\Gamma_m$  maxim tolerat

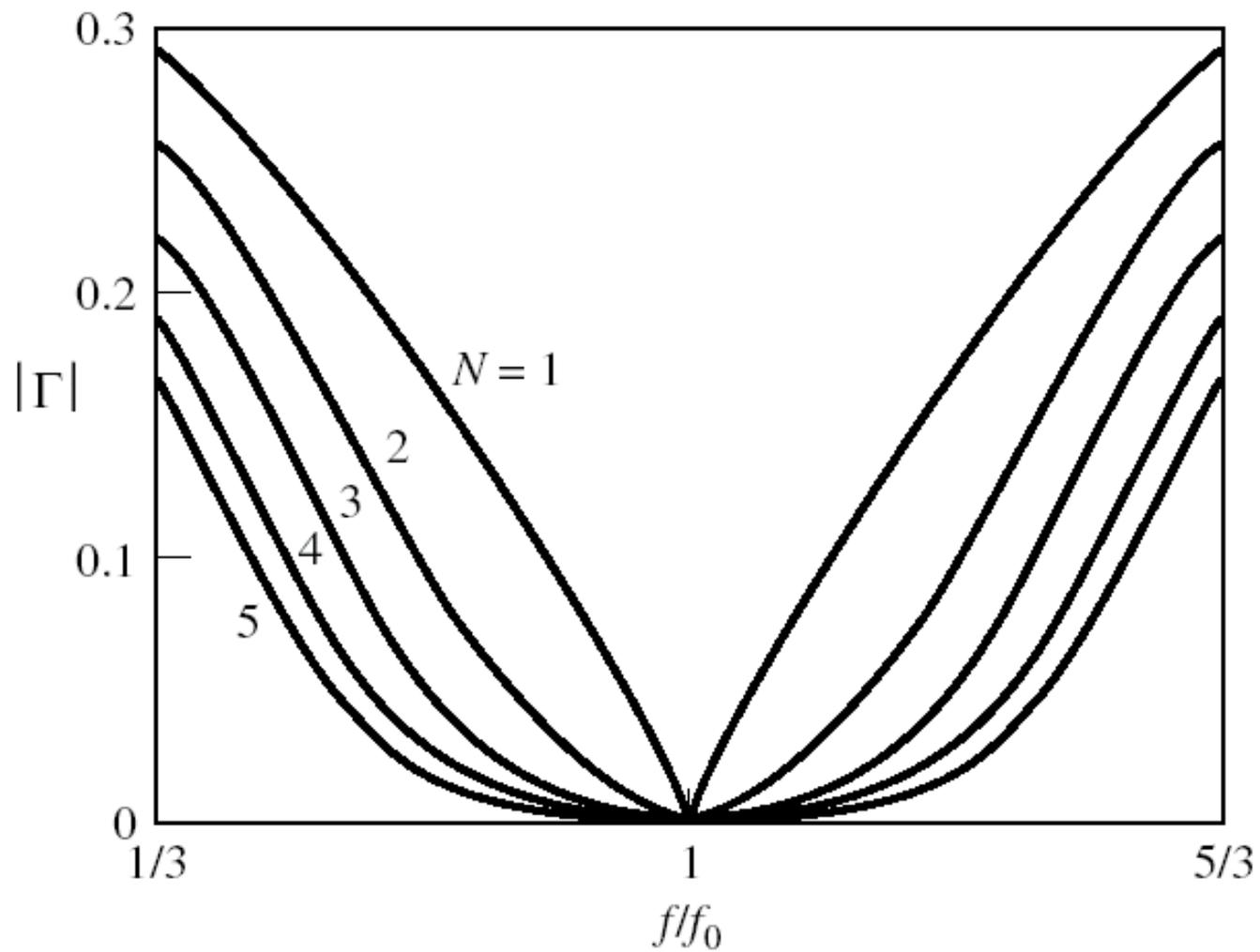
$$\Gamma_m = |\Gamma(\theta_m)| = 2^N \cdot |A| \cdot |\cos \theta_m|^N$$

$$\theta_m = \cos^{-1} \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$



$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$

# Banda



# Transformatoare cu mai multe sectiuni cu caracteristica binomiala rezultate exacte

$Z_L/Z_0$	$N = 2$		$N = 3$			$N = 4$					
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$		
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624		
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150		
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990		
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633		
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500		
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955		
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110		
$Z_L/Z_0$	$N = 5$					$N = 6$					
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_5/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_5/Z_0$	$Z_6/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

# Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de  $30\Omega$  la o linie de  $100\Omega$  la frecventa  $f_0=3\text{GHz}$ ,  $\Gamma_m=0.1$ 
  - $N = 3$

$$Z_L = 30\Omega \quad Z_0 = 100\Omega$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3! \cdot 0!} = 1 \quad C_3^1 = \frac{3!}{2! \cdot 1!} = 3 \quad C_3^2 = \frac{3!}{1! \cdot 2!} = 3$$

# Exemplu

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03\Omega$$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77\Omega$$

$n = 2$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87\Omega$$

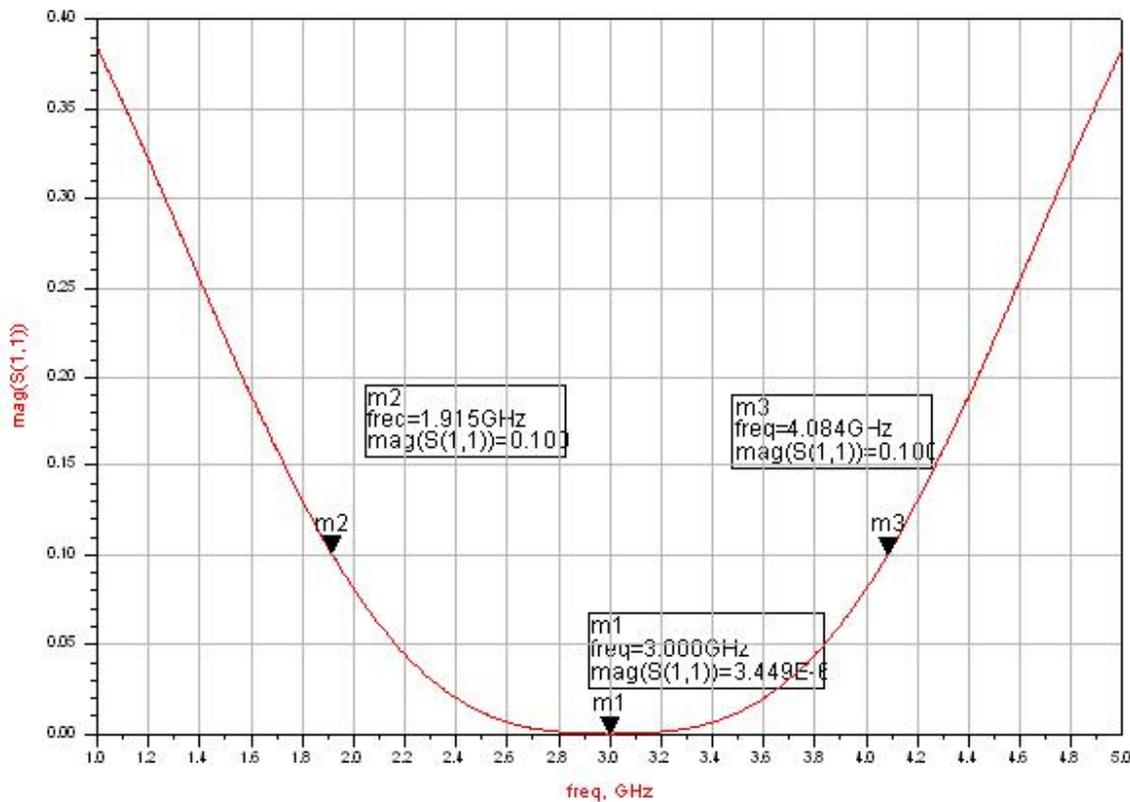
# Exemplu

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \arccos \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{1/N} \right] = 2 - \frac{4}{\pi} \arccos \left[ \frac{1}{2} \left( \frac{0.1}{0.07525} \right)^{1/3} \right] = 0.74$$

$$\Delta f = 2.22 \text{GHz}$$

# Simulare

## ■ Similar Lab. 1



$$\Delta f = 2.169 \text{GHz}$$

$$|\Gamma(3\text{GHz})| = 3.5 \cdot 10^{-6}$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev

- Raspunsul acestui transformator este de tip echiriplu in jurul frecventei de adaptare
- mareste banda in detrimentul riplului in banda de adaptare
- Se egaleaza functia  $\Gamma(\theta)$  cu un polinom Cebîșev

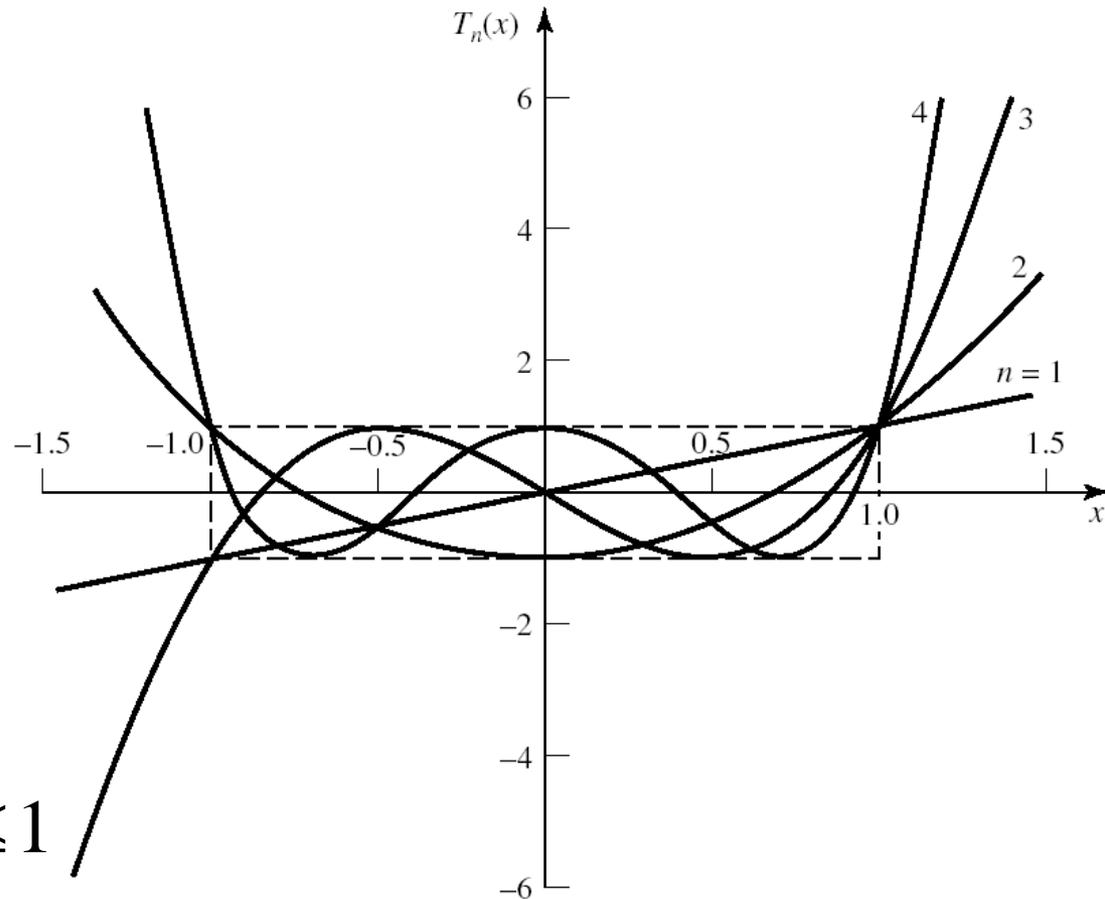
# Polinoame Cebîşev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



## ■ Echiriplu

$$-1 \leq x \leq 1 \Rightarrow |T_n(x)| \leq 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

# Polinoame Cebîșev

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \dots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:  $\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$

$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$

$$x = \cos \theta \quad |x| < 1$$

■ Se poate arata ca:  $T_n(\cos \theta) = \cos(n\theta)$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \quad T_n(x) = \cosh(n \cosh^{-1}(x)) \quad |x| > 1$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev

- Schimbare de variabila

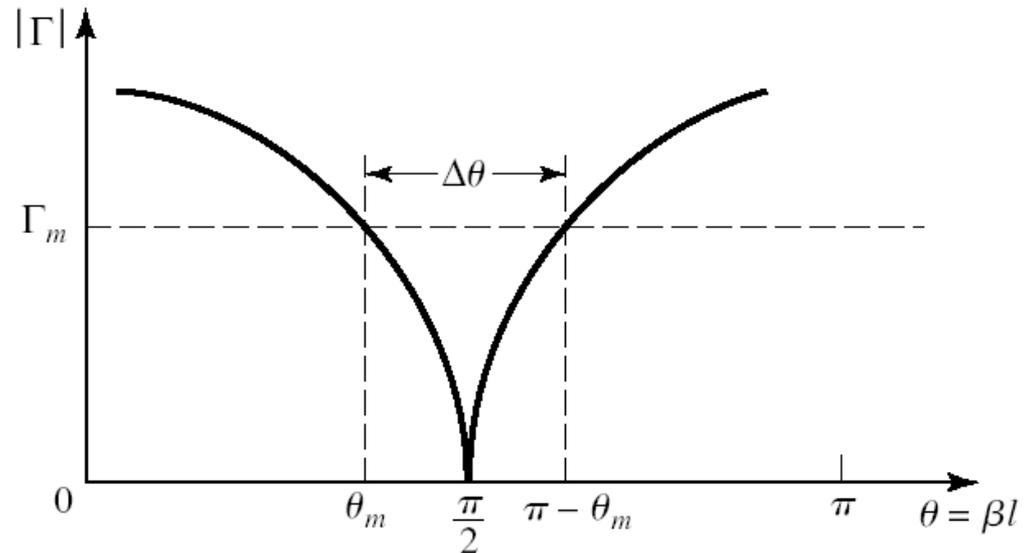
$$\theta = \theta_m \rightarrow x = 1$$

$$\theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos \theta}{\cos \theta_m}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$x = \sec \theta_m \cos \theta$$



# Transformatoare cu mai multe sectiuni de tip Cebîșev

$$T_1(\sec \theta_m \cos \theta) = \sec \theta_m \cos \theta$$

$$T_2(\sec \theta_m \cos \theta) = \sec^2 \theta_m (1 + \cos 2\theta) - 1$$

$$T_3(\sec \theta_m \cos \theta) = \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3 \sec \theta_m \cos \theta$$

$$T_4(\sec \theta_m \cos \theta) = \sec^4 \theta_m (\cos 4\theta + 4 \cos 2\theta + 3) - 4 \sec^2 \theta_m (\cos 2\theta + 1) + 1$$

- Cautam coeficientii pentru a obtine un polinom Cebîșev

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec \theta_m \cos \theta)$$

ultimul termen:

$$\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$
$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev

- $A, \theta \rightarrow 0$ , liniile de lungime 0, dispar

$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = A \cdot T_N(\sec \theta_m) \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad \Gamma_m = |A|$$

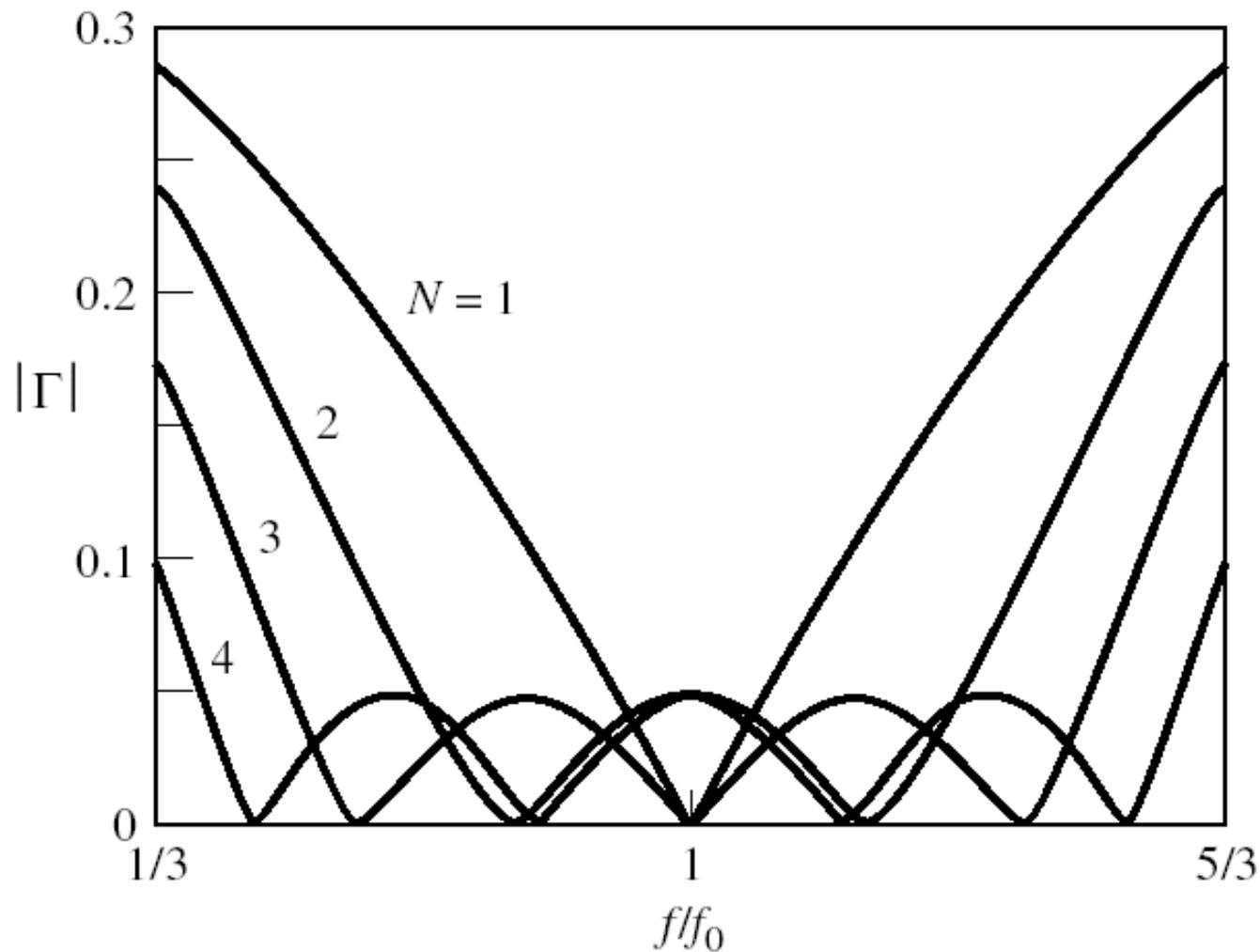
$$T_N(\sec \theta_m) = \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \cong \frac{1}{2\Gamma_m} \left| \ln \frac{Z_L}{Z_0} \right|$$

$$T_n(x) = \cosh(n \cosh^{-1}(x))$$

$$\sec \theta_m = \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \right) \right] \cong \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \left| \frac{\ln(Z_L/Z_0)}{2\Gamma_m} \right| \right) \right]$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

# Transformatoare cu mai multe sectiuni de tip Cebîșev



# Transformatoare cu mai multe sectiuni de tip Cebîșev

$Z_L/Z_0$	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

$Z_L/Z_0$	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

# Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de  $30\Omega$  la o linie de  $100\Omega$  la frecventa  $f_0=3\text{GHz}$ ,  $\Gamma_m=0.1$ 
  - $N = 3$      $Z_L = 30\Omega$      $Z_0 = 100\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta} [\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = Ae^{-j3\theta} T_3(\sec \theta_m \cos \theta)$$

$$|A| = \Gamma_m = 0.1 \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad Z_L < Z_0 \rightarrow A < 0 \quad A = -0.1$$

$$\sec \theta_m = \cosh \left[ \frac{1}{N} \cdot \cosh^{-1} \left( \left| \frac{\ln Z_L / Z_0}{2\Gamma_m} \right| \right) \right] = \cosh \left[ \frac{1}{3} \cdot \cosh^{-1} \left( \left| \frac{\ln(30/100)}{2 \cdot 0.1} \right| \right) \right] = 1.362$$

$$\theta_m = \arccos \left( \frac{1}{\sec \theta_m} \right) = 0.746 \text{ rad} = 42.76^\circ$$

# Exemplu

$$2[\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = A \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3A \sec \theta_m \cos \theta$$

$$\cos 3\theta \quad 2\Gamma_0 = A \sec^3 \theta_m \quad \Gamma_0 = -0.1263$$

$$\cos \theta \quad 2\Gamma_1 = 3A(\sec^3 \theta_m - \sec \theta_m) \quad \Gamma_1 = -0.1747$$

simetrie:  $\Gamma_3 = \Gamma_0; \quad \Gamma_2 = \Gamma_1$

# Exemplu

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353 \quad \Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega \quad \Gamma_1 = -0.1747$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

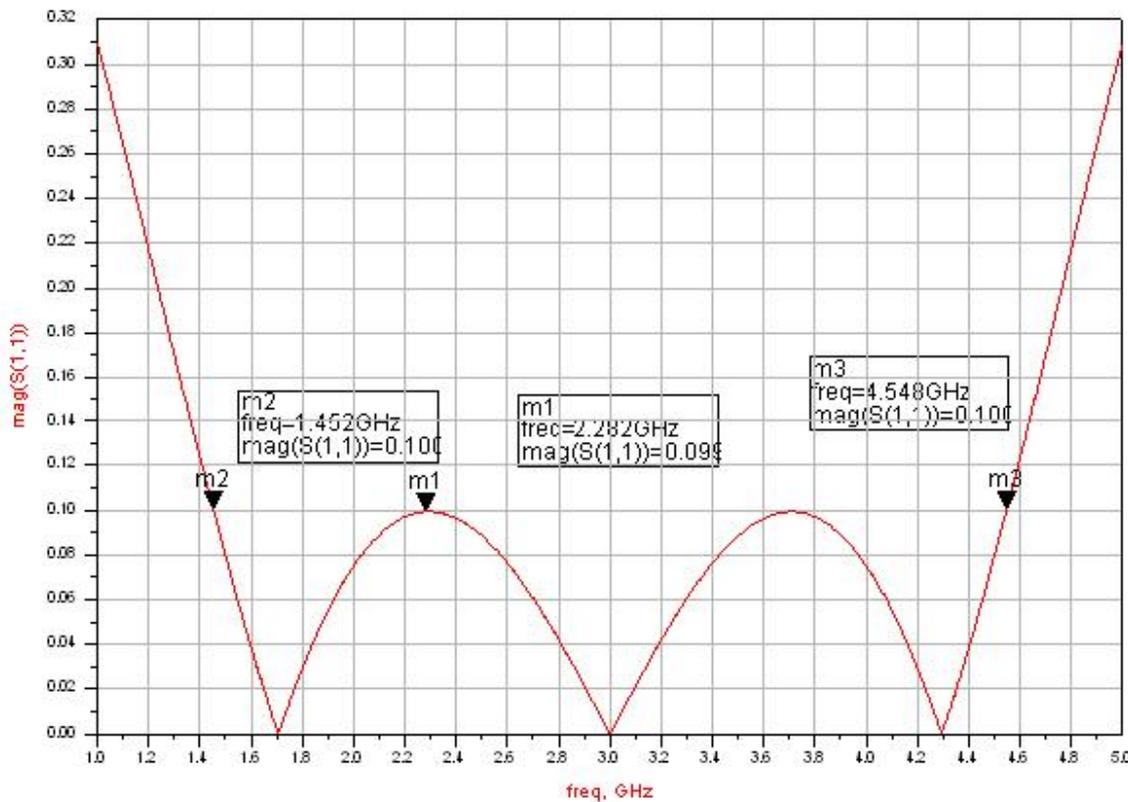
# Exemplu

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76^\circ}{180^\circ} = 1.045$$

$$\Delta f = 3.15 \text{GHz}$$

# Simulare

## ■ Similar Lab. 1



$$\Delta f = 3.096GHz$$

$$|\Gamma(3GHz)| = 4.17 \cdot 10^{-5}$$

$$|\Gamma(2.282GHz)| = 0.09925$$

# Solutii exacte

- G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House Books, Dedham, Mass. 1980

Lab. 1

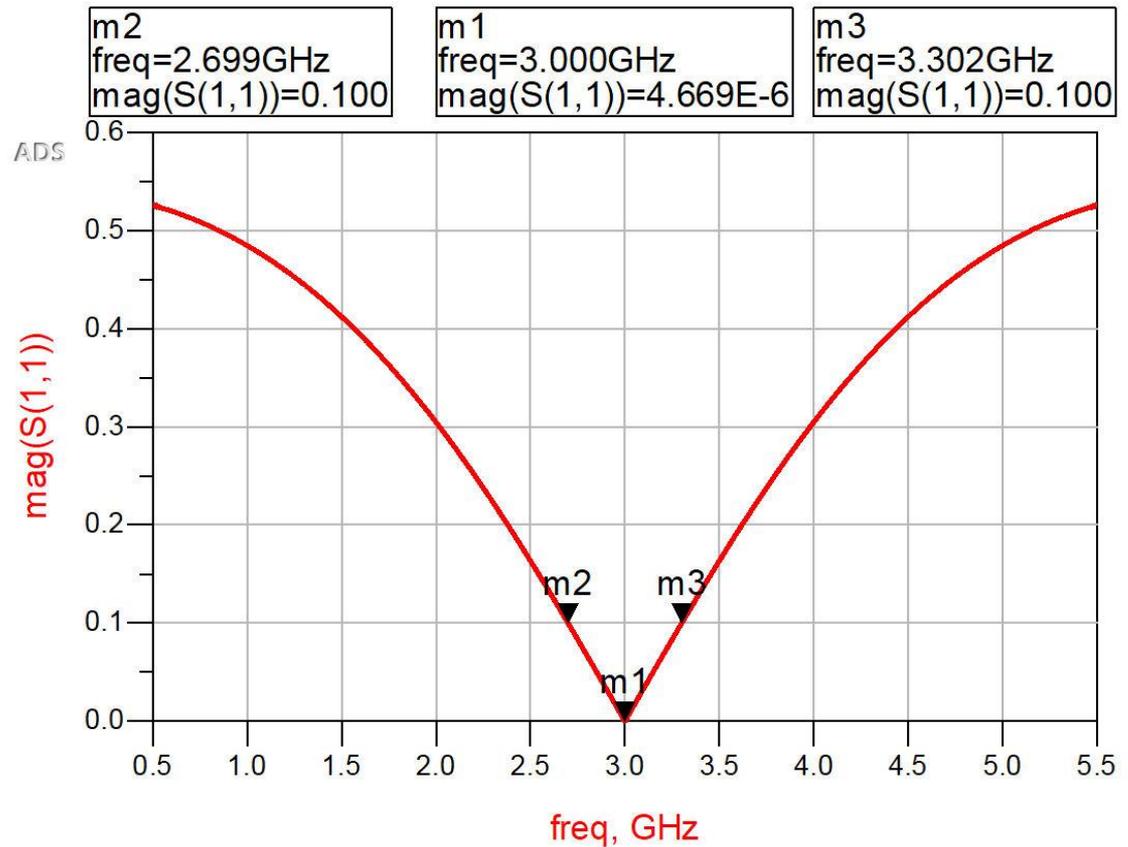
# Adaptarea de impedanță

# Sfert de lungime de unda

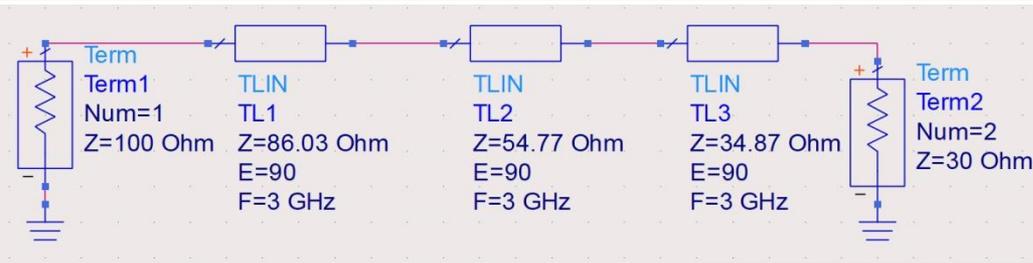


 S-PARAMETERS

S\_Param  
SP1  
Start=0.5 GHz  
Stop=5.5 GHz  
Step=0.001 GHz

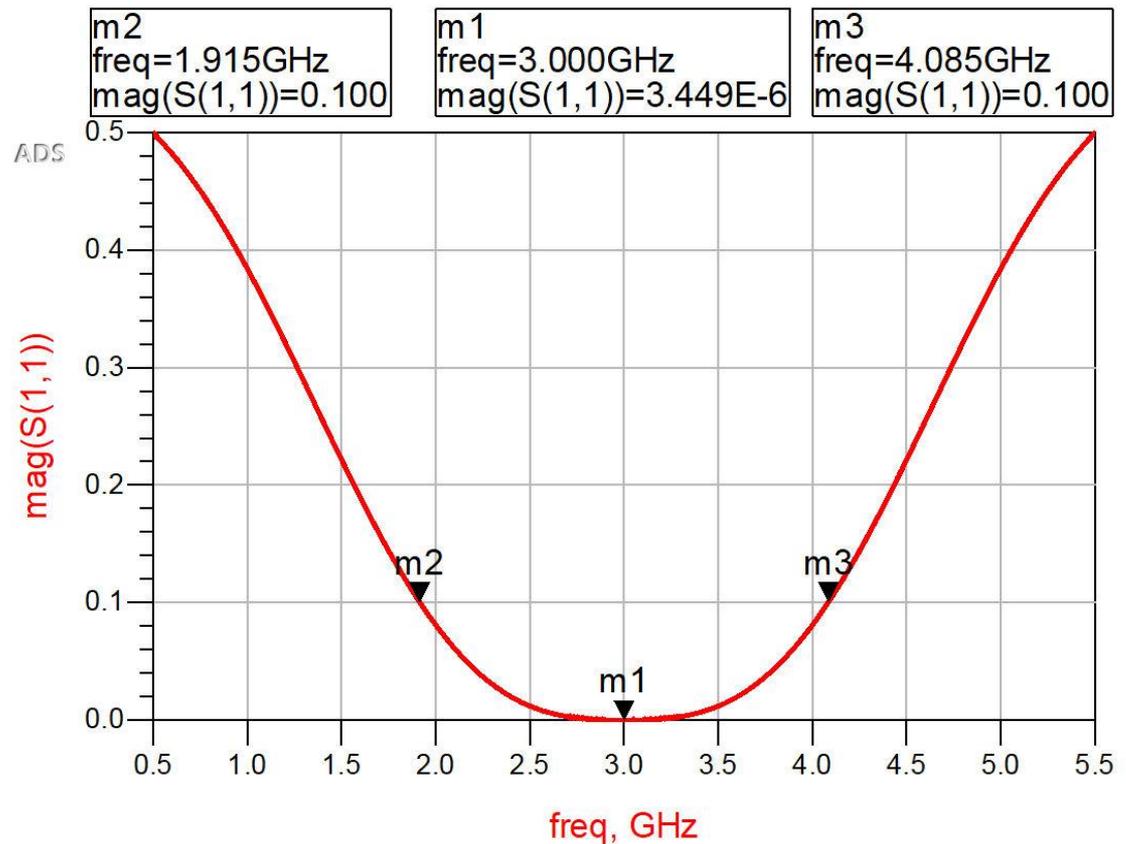


# Binomial

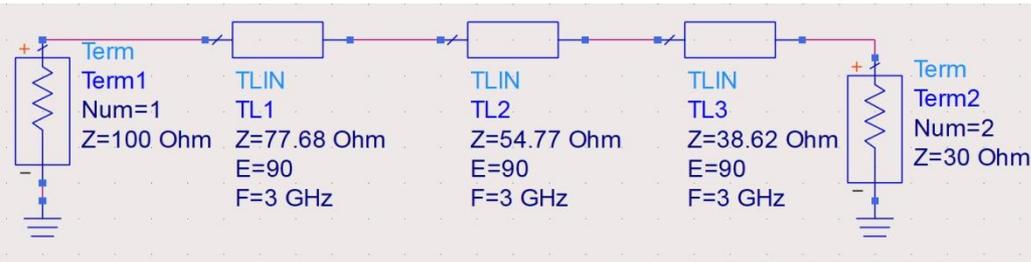


 **S-PARAMETERS**

S\_Param  
SP1  
Start=0.5 GHz  
Stop=5.5 GHz  
Step=0.001 GHz

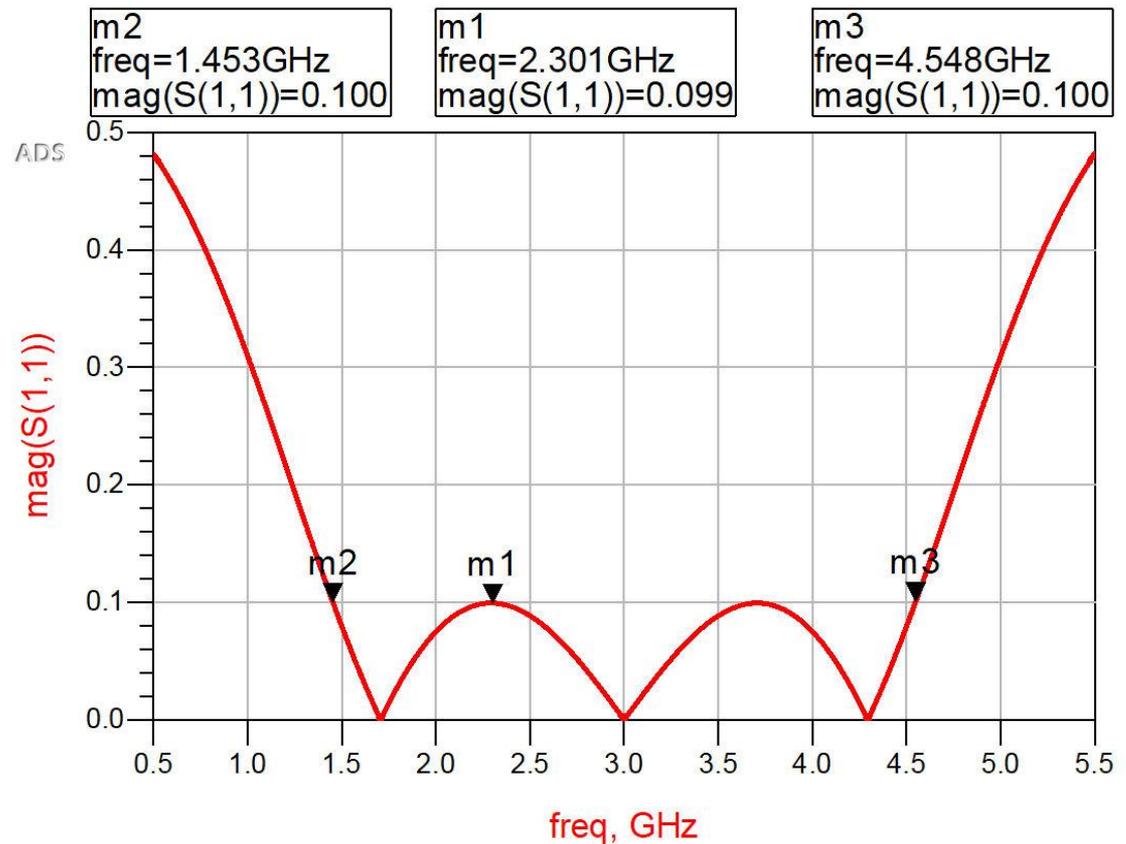


# Cebâşev



## S-PARAMETERS

S\_Param  
SP1  
Start=0.5 GHz  
Stop=5.5 GHz  
Step=0.001 GHz



# Contact

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